

A N
INTRODUCTION

The 1st To Hunt 1825
Natural Philosophy:

OR,
Philosophical Lectures

Read in the
University of Oxford,

Anno Dom. 1700.

To which are Added,

The DEMONSTRATIONS of Mon-
sieur HUYGENS'S *Theorems*, concerning
the Centrifugal Force and Circular Motion.

By **JOHN KEILL M. D.**
Savilian Professor of Astronomy. F. R. S.

Translated from the last Edition of the Latin.

THE FOURTH EDITION.

L O N D O N:

Printed for M. SENEX, W. INNYS, T. LONGMAN and
F. SHEWELL. M DCC XLV.

INTRODUCTION

Wm. L. G. (op. p.)

Philosophical Transactions

38.
2. 8.
82



IN THE YEAR 1800

the Cambridge and London

BY JOHN W. L. G.

of the University of Cambridge

Trinity College, Cambridge

The London

and Cambridge

W. L. G. W. L. G. W. L. G.

Cambridge and London



THE
PREFACE.

ALTHOUGH now-a-days the Mechanical Philosophy is in great Repute, and in this Age has met with many who cultivate it; yet in most of the Writings of the Philosophers, there is scarce any thing Mechanical to be found besides the Name. Instead whereof, the Philosophers substitute the Figures, Ways, Pores, and Interstices of Corpuscles, which they never saw; the intestine Motion of Particles, the Collussions and Conflicts of Acids and Alkalies, and the Events that thence arise, they relate so exactly, that there is nothing but a Belief wanting in the History of Nature, as often as they set forth the Miracles of their subtle Matter: I say, Miracles, for certainly that must be a sort of a Miracle, which happens contrary to the well-known Laws of Nature, and the established Principles of

Mechanicks ; as would be all the Phenomena of Nature, if they were produced by a subtile Matter, and the Method of Operation that is delivered by the Philosophers.

To explain the Works of Nature, they make use of such Postulata, as can neither be conceded, or understood ; and which are indeed more complicated than the Phenomena themselves, whose Causes they investigate. Besides, if their Postulata are granted yet the Effects cannot be thence deduced, whose Causes and Origin they boast to have discovered.

BUT that nobody may suspect that this is rashly and malevolently asserted by us, we shall here examine the Theory they have framed to explain the most universal Affection of all Terrestrial Bodies ; I mean Gravity, which they principally boast of having been able to deduce by Mechanical Laws from the Action of a subtile Matter.

THE Cartesians will have Gravity to arise from the Action of a Celestial Matter, which being agitated in a Vortex, is carried about the Earth, and consequently recedes, as much as it can, from the Earth, and forces Terrestrial Bodies being less agitated towards the Earth. Or, as the Moderns have more clearly explained the Opinion of Cartes, since the ethereal Matter makes continual Gyration about the Earth, it will have, after the manner of Bodies that move circularly, an Endeavour of receding from the Center of Motion, so that the Terrestrial Bodies having a less Force, will be compelled by it towards the Center ; as Water gravitating towards the Earth, drives upwards or towards the Circumference, Bodies of a less Weight in respect to their Bulk, which are immersed in it.

T H I S,

The PREFACE.

v

THIS, however specious it may seem at first appearance, if it be brought to an Examination, will be found contrary to almost all the Laws of Nature. For in the first place, the Cartesians require their Ethereal Matter to be carried round the Earth in Circles; but from what Cause that Motion arises, or by what Means it is continued, is as difficult to explain, as to give a Reason for Gravity itself. They therefore who contend that Gravity proceeds from thence, undertake to explain what is unknown by what is still more unknown, especially since there may be produced divers Arguments, whereby that Rotation is wholly destroyed. But let us grant to the Cartesians their Postulum, and then let us see, whether this Phenomenon does thence follow. Since it is necessary that the Velocity of the Vortex revolving about the Earth, is, at the Superficies of the Earth, equal to the Velocity of the Earth's Rotation, (for if it was greater, some part of the Motion would be communicated to the Earth; whence it would happen, that its Velocity would be always diminished, and that of the Earth increased, till they arrived at an Equality) so that from the Magnitude of the Earth, and the Time of its Rotation being known, there will be given the Space, which a Body, urged by the Centrifugal Force of the Celestial Matter, can run over in a given Time; namely, that which is equal to the Square of the Arch described in the mean while, applied to the Diameter of the Circle: By Lemma 2. in the Demonstration of Mons. Huygens's Theorems of the Centrifugal Force and Circular Motion. If a Calculation be made from this Principle, it will be found that the Space which will be run over in a Second of Time, by a Body agitated by a Centrifugal Force of the *Æther*, will not ex-

ceed half a Foot: If therefore the Effect of Gravity was produced mechanically, heavy Bodies in a Second of Time would not descend above half a Foot; but heavy Bodies in their Motion downwards will pass over in the same Time fifteen Feet. So that if after this manner the Æther was the Cause of Gravity, it would act contrary to the Laws of Mechanicks, by making a Body descend through fifteen Feet in one Second of Time.

To avoid the Force of this Objection, the Cartesians suppose the Rotation of the ethereal Matter to be much swifter than the Rotation of the Earth. Which, though it is impossible, yet if we also concede to them this, there will not thence proceed the Mechanical Action of Gravity. For since the Matter of the Vortex is always carried in Circles parallel to the Equator, and the Directions of the Centrifugal Forces are always made in Lines lying in the Planes of these Circles; it follows, that all Bodies must descend in these Planes, and tend perpendicularly to the Axis, and not to the Surface of the Earth. If therefore the subtil Matter acted mechanically, it would force Bodies perpendicularly to the Axis; whence since, according to these Theorists, it compels them to tend to the Center of the Earth, it produces an Effect contrary to the true Laws of Mechanicks.

THAT they may remove this Difficulty, they farther suppose the ethereal Matter to be carried not in Circles parallel to the Equator, but in great Circles of a Sphere. But how it is possible to conceive this, I am at a loss to know; for since every great Circle twice intersects all the other Circles that are infinite in number, it is necessary that the Motion of any Particle should be hindered by infinite
others

others moving different ways, and at length its Motion should cease, if at first an equal Quantity of Motion was impressed on all the Parts; or that, lastly, it should be all carried in parallel Circles, if at the beginning the Motion was greater towards one Part than the other. Moreover, it may be asked, whence it comes to pass that the ethereal Matter does move in the Superficies of the outermost Sphere, since it has a Centrifugal Force, it seems as if it should recede from thence; what then is it that hinders it from doing so? They are wont to answer, That the ambient Bodies compress the Matter in the exterior Sphere, and prevent its escaping. But since this Matter must press other Bodies encompassing it, it is necessary that it communicates a Motion to them, and these Bodies in like manner will impress a Motion on others encompassing them, and so the Motion of the subtle Matter will be propagated in infinitum; whence of necessity its Celerity must languish by degrees.

THERE are many other Difficulties which accompany these Mechanical Explications of Gravity; one whereof we shall here propose, and which extends to all the Theories of this sort. And it is this: If a Body is after any manner driven downwards by a subtle Matter, the Force whereby it is driven, will be necessarily as the Number of Particles, by the acting of which together, the Body is constrained to tend towards the Earth: but the Number of Particles is as the Superficies of the Body; wherefore the Force whereby the Body is pressed downwards, will be as its Superficies, and not as the Quantity of Matter in the Body, which is contrary to Experience. Nor will all the rest of the Hypotheses, which they frame to account for the appearance of

other things, be found less repugnant to the Laws of Nature, if they are brought to an Examination.

ALL these Errors seem to spring from hence, that Men ignorant of Geometry presume to philosophize, and to give the Causes of Natural Things. For what can we expect but Mistakes, from such, as having neglected Geometry; the Foundation of all Philosophy, and being unacquainted with the Forces of Nature, which can only be estimated by the means of Geometry, do yet attempt to explain its Operations, by a Method not at all agreeing with the Rules of Mechanicks?

AMONGST the Philosophers of this sort, Cartes leads the Van; who though he was a famous Geometer, yet that he might accommodate himself to the idle and common Herd of Philosophers, made no use of Geometry in his Philosophy; and although he pretended to explain all things mechanically by Matter and Motion, yet he introduced a Philosophy which was as contrary to the true Laws of Mechanicks, as was possible. Those may be reckoned of his Sect, who refuse the Labour of Philosophizing aright, that is, Geometrically: And such are by much the greatest number, being scattered almost over the whole Earth.

BUT although so great a part of the Philosophers have embraced the Shadow of Philosophy, and not its Substance; yet there have not been wanting (nor, I hope, ever will) such as have employed real Pains in discovering the true Laws of Nature, and in investigating from thence the Causes of Things by Mechanical Principles.

AMONGST the antient Philosophers, the Divine Archimedes is of the greatest Eminence; who, besides his Geometrical Labours, has left us the Principles of Mechanicks

nicks and Staticks demonstrated in two Books de Aequi ponderantibus & de Humido Insidentibus. After him through a long Series of Years the Mechanical Philosophy lay in Obscurity, nor was it cultivated except by a very few Persons of a more refined Understanding. Amongst whom we may justly reckon Roger Bacon of Oxford, and Cardan. But at length, about the beginning of the last Age, that noble Lyncean Philosopher Galileo, having by the means of Geometry penetrated into the Secrets of Nature, framed a new Science of Motion, and shewed a Method whereby the Mechanical Causes of Things might be discovered. Then the Famous Torricelli and Paschal treading in his very Footsteps, improved Philosophy by new Speculations. And lastly, the Societies of London and Paris were instituted by two powerful Monarchs, for the Advancement of Philosophy; whereby the Knowledge of Nature has been increased with wonderful Inventions, not only such as terminate in bare Speculation, but with many others which are of common Use to Men. It would be a difficult Task to reckon up the innumerable Benefits that accrue to Mankind from the Labours of both Societies: nor is it an easy matter to shew, how much succeeding Ages will be obliged to the Geometrical Demonstrations of the Illustrious Monsieur Huygens, concerning the Motion of Pendulums; or to the curious Experiments of the Honourable Mr. Boyle, whereby he has disclosed many wonderful Secrets of Nature. Our late Posterity will peruse with a grateful Mind Dr. Wallis's Treatise of Motion; a Work the most perfect of its kind. The Philosophers will now no longer be troubled to find out the Causes of Rivers and Winds, since
the

they have been delivered by that acute Geometer Dr. Halley, tho' before him they have been sought after in vain.

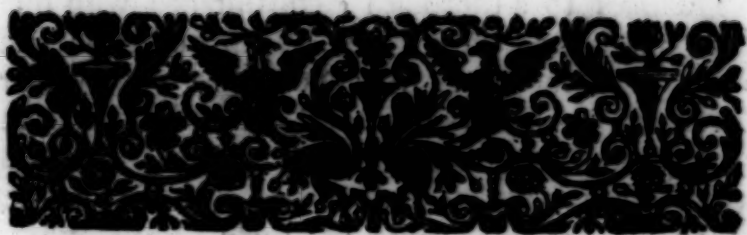
I SHOULD proceed in enumerating the Merits of others towards the real Philosophy, if I did not find myself obliged to stop, to mention the great Inventions of Sir Isaac Newton, whose prodigious Genius has laid open more and abstruser Mysteries of Nature, than Men could ever have hoped for: but since it is impossible to comprehend his Discoveries within the narrow Limits of this Preface, we shall only undertake to say thus much, That what all our Predecessors from Time immemorial have handed down to us concerning the Mechanical Philosophy, does not amount to the tenth part of those Things, which Sir Isaac Newton alone, through his vast Skill in Geometry, has found out by his own Sagacity. But how the Mechanical Principles may be easily applied to explain the Affections of such Bodies as are at a distance from us, namely, the Motions and Inequalities of the Planets; the learned World has been lately informed by the Philosophical and Geometrical Elements of Astronomy published by Dr. Gregory, the Savilian Professor of Astronomy: A Work that will last as long as the Sun and Moon endure.

SINCE therefore such is the State of the Mechanical Philosophy, that there is no Admittance to it but by the means of Geometry; some of my Friends have requested of me, that I would undertake to explain to the Youth of this University its easiest Principles, and such as only depend on the first Elements of Geometry; which thing was also asked of me in the most obliging manner in the world, by the
Gentleman

Gentleman who substituted me in the publick Schools for the undertaking this Work. And this was Sir Thomas Millington M. D. Sedleian Professor of Natural Philosophy in this University, and President of the College of Physicians in London; a Person adorned with Learning of every kind. By his Advice it was, that I read the following Lectures in this University: And in them my principal Care was, that Beginners might have clear and distinct Conceptions of the general Affections of Bodies; for all the Errors in Philosophical Matters have their Origin from the obscure and false Ideas of Things: so that I have endeavoured, as much as I was able, to expound clearly the Extension, Solidity, and Divisibility of Body, which have been delivered by others very obscurely. And then I have explained the Nature and Properties of Motion, which may be easily conceived by all unless some sort of Philosophers; and I have thence deduced the Laws of Nature, and have shewn that the Force of Gravity, or the Weight of Bodies, is proportionable to the Quantity of Matter in those Bodies, and the Principle whereby, thro' the means of Engines, great Weights are raised. Next, I have made manifest the Laws of Motion, and the Cause of the Acceleration of heavy Bodies depending on them; and by what Proportion the Spaces passed over by heavy Bodies, increase or decrease according to the different Intervals of the Times. To these succeed the Rules of Congress, as well in hard as elastick Bodies, and the Manner whereby the Magnitude of a Stroke may be estimated: to which I have joyned the Compositions and Resolutions of Motions, and some other Theorems, of no small use in Philosophy. And that the Philosophers may farther perceive, how far even the

the Use of the Elementary Geometry extends itself in the Knowledge of Natural Things, I have demonstrated from the Elements the very beautiful Theorems of Mons. Huygens concerning the Centrifugal Force and Circular Motion.





LECTURE I.

Of the Method of Philosophizing.

S it is our Purpose in these Lectures to endeavour to deliver the Explication of the Properties and Affections of Bodies, we have thought it not improper, in the first place, to say something concerning the Sects, and Principles, and Methods of the Philosophers; and at the same time, to declare our own Manner of Proceeding, in the investigating the Causes of Natural Things.

AMONGST the various Sects of Philosophers that have wrote on Physical Subjects, there seems to have been four of the greatest Eminence. The first were those who attempted to illustrate, (shall I say?) or rather conceal, the Natures of Things, by the Properties of Numbers and Geometrical Figures. Such were the *Pythagoreans* and the *Platonists*; for these not being willing that their Opinions should be expos'd to the common People, did therefore, as it were, cast a Vail over them, by the means of Images and

B

Micro-

Hieroglyphicks, that they borrowed from Geometry and Arithmetick: nor indeed did they admit their own Disciples to a thorough Acquaintance with their real Opinions and the greatest Depths of their Philosophy, until they had first undergone many Years of Probation and Trial. Tho' this Method seems extremely well calculated to keep up the Dignity of their Profession, yet it had a very ill effect in respect to us, the Successors of these Philosophers. For by this means their Opinions have been handed down to us with so much Obscurity and Darkness, that it is almost impossible to discern, thro' the Disguises that they wrapped them up in, what were their true Sentiments concerning the Natures of Things. But however, notwithstanding the Philosophy of this Sect appears very mysterious to us, yet we may be certain, from their Manner of delivering it, that they looked upon Geometry and Arithmetick to be absolutely necessary to solve the Phenomena of Nature, and that they used them to that very purpose.

THE second sort were the *Peripateticks*. This Sect explained their Philosophy by Matter and Forms, Privations, Elementary Virtues, occult Qualities, Sympathies and Antipathies, Faculties, Attractions, and the like. But these Philosophers, in my Opinion, seem not so much to have discovered the Causes of Things, as to have given proper Names to the Things themselves, and to have invented such Terms, as are very fit to express natural Actions.

THE third Order of Philosophers are such as proceed upon *Experiments*: and these make it their sole Business, that the Properties and Actions of all Bodies may be manifested to us, by the means of our Senses. And indeed Philosophy has received very considerable Advantages from their Labours; and it might thereby have been still further improved, if the Expe-

Lect. I. *Natural Philosophy.*

3

Experimenters themselves had not too often distorted their Experiments and Observations, in order to favour some darling Theories they had espoused.

THE last sort of Philosophers are such as are wont to be called *Mechanical*. They that lift themselves under this Banner, imagine they can explain all the Phenomena of Nature by Matter and Motion, by the Figure and Texture of the Parts, by subtle Particles, and the Actions of Effluvia; and they likewise contend, that these Operations are brought about by the known and established Laws of Mechanicks.

AMONGST these various Ways of Philosophizing, as there is no particular one, wherein we do intirely acquiesce; so in each, there are some things which we can approve of. Wherefore we shall chuse out of all of them what may be thought useful, and thence compose the Method we shall here follow.

AND first of all, in imitation of the ancient *Pythagoreans* and *Platonists*, we shall call in to our assistance Arithmetick and Geometry, as Arts very necessary to Philosophy, and without which, but little of Certainty can be ever discovered in natural Causes. For since every Physical Action depends upon Motion, or at least it cannot be performed without Motion; the Quantity and Proportion of Motion, the Magnitude, Figures, Number, and Collisions of Bodies in motion, and their Forces necessary to move other Bodies, ought to be investigated. But it is impossible to determine all these, without being instructed in the Nature of Quantity and Proportion: and therefore there is occasion for those Arts, that demonstrate their Properties; and consequently Geometry and Arithmetick must be thought necessary to any one, that would make any progress in Natural Philosophy.

SECONDLY, we shall not be ashamed to use, with the *Peripateticks*, the Terms *Quality*, *Faculty*, *Attraction*, and the like; not that by these words we pretend to define the true and physical Cause and Modus of Action, but as these Actions may be augmented and diminished, and therefore since they have the Properties of Qualities, the same Name may not unfitly be applied to them, so that we thereby only mean to express the Ratios of the Forces or their Augmentation and Diminution. For example, we may say that Gravity is a Quality, whereby all Bodies are carried downwards, whether its Cause arises from the Virtue of the central Body, or is innate to Matter itself; or whether it proceeds from the Action of the *Æther* agitated by a centrifugal Force, and so tending upwards; or lastly, whether it is produced after any other manner whatever. So likewise we may call the Endeavour of Bodies to approach one another, Attraction; by which word we do not mean to determine the Cause of that Action, whether it proceeds from the Action of the Bodies tending mutually to one another, or from their being agitated by Effluvia emitted, or from the Action of the *Æther*, or Air, or any Medium whatever that impels the Bodies, which float in it, towards one another: such Actions, I say, we may express by these Words. And if the true Causes are hid from us, why may we not call them occult Qualities? Certainly by the same right, as in Algebraical Equations we denote the unknown Quantities by the Letter x or y ; and not by a very unlike Method we may investigate the Intentions and Remissions of these Qualities, which follow from some certain supposed Conditions. We will illustrate this by an Example.

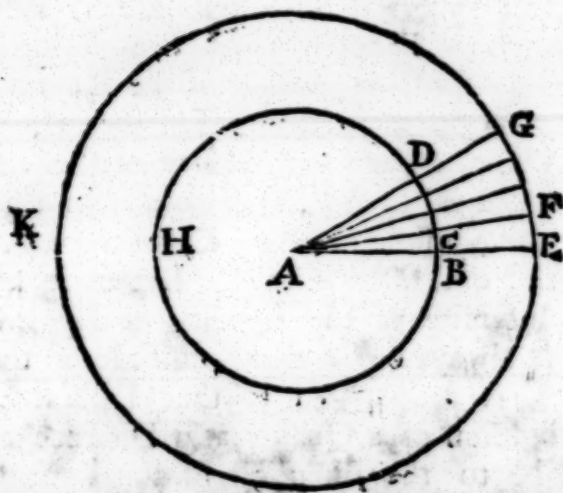
• HOWEVER ignorant we are of the Nature of Qualities, and how much soever the Modus of Operations is concealed from us, we are able notwithstanding

Lect. I. *Natural Philosophy.*

5

standing to demonstrate the following Theorem concerning their Intension and Remission. Every Quality or Virtue that is propagated every way in right Lines from a Center, is diminished in a duplicate Proportion of the Distance from that Center.

LET A be a Point or Center, whence every way the Quality is diffused, in the right Lines AB , AC , AD , and innumerable others spread indefinitely through the whole Space. I say, that the Intension of the Quality decreases in a Ratio duplicate of that, whereby the Distances increase; or, which is the same thing, its Intension at a Distance equal to AB is to its Intension at a Distance equal to the right Line AE , reciprocally in a duplicate Ratio of the Distance AB , to the Distance AE ; that is, directly as the Square of AE to the Square of AB .



Since, from the Hypothesis, the Quality is propagated in a Sphere every way by right Lines, its Intension at any Distance from the Center, will be proportionable to the Spissitude or Density of the Rays at the same Distance. By Rays I here mean the rectilineal Ways by which the Quality is diffused: Now the Rays that at the Distance AB are diffused thro'

the spherical Superficies BCDH, will be dispersed at the distance AE, thro' the whole spherical Superficies EFGK: but the Spissitude of any given Rays are reciprocally as the Spaces they take up; namely, if the Superficies EFGK is double the Superficies BCDH, the Rays at the Superficies BCDH, will have double the Density, that the same Rays will have at the Superficies EFGK: and if the Superficies EFGK is triple the Superficies BCDH, the Rays likewise at the Superficies BCDH will have triple the Density that the same Rays will have at the Superficies EFGK; and universally whatever proportion the Superficies EFGK has to the Superficies BCDH, the same proportion will have the Density of the Rays at the Superficies BCDH to the Density of the same Rays, at the Superficies EFGK. And from *Archimed. of the Sphere and Cylinder*, it appears that all spherical Superficies are in a duplicate Ratio of their Diameters or Semidiameters: the Spissitude therefore, or Density of the Rays by which the Quality is propagated to a Distance equal to the Distance AB, will be to the Density of the same Rays at a Distance equal to AE, reciprocally in a duplicate Ratio of the Semidiameter or Distance AB, to the Semidiameter or Distance AE. But as it has been said, the Intension of the Quality at any given Distance, will always be as the Spissitude of the Rays by which it is propagated to that Distance; therefore the Intension of a Quality at a Distance equal to AB, will be to the Intension of the same Quality at a Distance equal to AE, reciprocally in a duplicate Ratio of the Distance AB to the Distance AE; that is, directly as the Square of AE to the Square of AB.

WE have demonstrated this Theorem universally, whatever is the Nature of the Quality, so that it acts in right Lines; and it hence follows, that the
 .Inten-

Lect. I. *Natural Philosophy.*

7

Intensions of Light, Heat, Cold, Perfumes, and the like Qualities, will be reciprocally as the Squares of their Distances from the Point whence they proceed. Hence also may be compared amongst themselves, the Actions of the Sun on different Planets: but this is not the Business of our present Design.

AFTER the Ratios of the Forces are discovered in given Conditions or Suppositions, then those Ratios are to be compared with the Phenomena of Nature, that there may be found out, what Conditions of the Forces belong to every sort of Bodies. But in order to obtain this, many of the Experiments that the third Sect of Philosophers have delivered down to us, must be made use of: tho' this ought not to be done without great Caution; for we are well apprised how fond these Gentlemen are of their Theories, how willing they are that they should be true, and how easily they deceive both others and themselves, in trying their Experiments. Such therefore as are produced by all, and which succeed upon every Trial, we receive as undoubted Principles or Axioms: as likewise we ought sooner to give credit to those Experiments that are more simple and easy to be shewn, than to those that are more compounded, and difficult to be performed.

LASTLY, We ought to inquire with the antient Atomists, and the Followers of the new Philosophy, what are the Phenomena that may be explained by Matter and Motion, and the known and established Laws of Mechanicks.

BUT that we may proceed in this Affair with the greater safety, and, as much as possible, avoid all Errors: we shall endeavour to observe the following Rules.

FIRST, We shall, after the Method of the Geometers, premise such Definitions, as are necessary to arrive at the Knowledge of Things. It is not to

be expected that these should be logical Definitions, which consist of the *Genus* and *Difference*; or such as discover the intimate *Essence* and ultimate Cause of the thing defined: these Matters I leave to be disputed by others; for ingenuously to confess my own Ignorance, the intimate Natures and Causes of Things are not known to me. And whatever acquaintance I have with Bodies or their Actions, I obtained it either by the help of my Senses, or else deduced it from some of their Properties, which Properties were discovered by the same means. I shall therefore, instead of such a Definition as the Logicians are wont to give, exhibit a Description; whereby the thing described may be clearly and distinctly conceived, and likewise be distinguished from every thing else. Wherefore we shall define all things by their Properties, chusing out one or more of the simplest, which by Experience we are certain do really belong to the things themselves; and then from these, we shall after a Geometrical manner deduce other Properties of the same things. Many modern Philosophers have offended against this Rule; who define things not by any Properties which do certainly belong to them, but by *Essences* and *Natures* which they suppose to be in the things themselves. They suppose it indeed, but at the same time it does not at all appear, whether such Natures as they define, are really in the things themselves. The *Cartesians*, for example, say that a Fluid has its Parts in continual Motion, but it does not appear from our Senses, or Experience, or Reason, that such is the Nature of a Fluid: nay, that very Argument which they produce to confirm this their Hypothesis, we shall overthrow by a Geometrical Demonstration. For they will have it, that the Resistance of a Body moving in a Fluid, is less, if the Parts of that Fluid are agitated by an intestine Motion, than if there was

Lect. I. *Natural Philosophy.*

9

was no such Motion in the Fluid; the contrary to which, we shall demonstrate, when we come to treat of the Resistance of Fluids.

BUT the Writers of the Mathematical Philosophy have followed a much better course, who take their Definition of a Fluid from its most obvious Property. A Fluid, they say, is a Body whose Parts yield to the smallest Force or Impression, and by receding, are easily moved in respect to each other. From which Definition they deduce many beautiful Theorems that are of Service in the Occasions of Life; whereas the *Cartesian* Philosophers from theirs have produced nothing either certain or solid, much less useful.

SECONDLY, While we are investigating natural Truths, it will be of advantage to consider only the Conditions that were supposed at first, abstracting for a time from all other Considerations whatever. For as the human Mind is finite, so if it be disturbed by too great a number of things at once, it will by that means be rendered incapable of making any Discoveries. The Mechanical Writers have observed this Rule in their comparing the Spaces passed over by two Bodies in motion: for they look upon the Bodies in this case as Points, abstracting from the Consideration of their Magnitude, Figure, and Colour, which make no alteration at all in respect to the Length they have moved.

THIRDLY, It is necessary to begin with the most simple Cases first; and having once settled them, we may thence advance to such as are more compounded. So the same Mechanical Philosophers at first suppose the Motion of Bodies to be *in vacuo*, or in a Medium that has no Resistance; and having determined the Laws of Motion in that Case, they thence proceed to investigate the Laws of Resistance, and lastly to discover what Changes are thereby likely to arise to
Bodies

Bodies in motion. For the less the Bodies are resisted by the Medium they move in, the less will the Laws of the Motion of Bodies in that Medium differ from the Laws before found out. So likewise in Hydrostaticks, we consider the Fluid as having no Tenacity or Coherence of its Parts, but that they are separable by the least Force whatsoever: and it is on this supposition, that we determine the Positions of Bodies immersed in such a Fluid, and the Pressures of the Fluid on such Bodies. Tho' perhaps there is not in Nature such a Fluid, as has no Cohesion of its Parts; wherefore the Variation or Disagreement in the Laws of a Fluid whose Parts do cohere, from those of a Fluid which we suppose to have no Cohesion in its Parts, ought to be investigated; and if that Tenacity or Coherence of Parts in the Fluid be but small, the Disagreement from the Laws first discovered, will be but small, and scarce discernible.

MOST of the Theorists offend against this Rule; who neglecting, or not thoroughly understanding the first and more simple Principles of the mechanical Philosophy, at the very first stroke attempt the most difficult Problems, and rashly enough endeavour to shew how a World, a Planet, or an Animal might be formed. These are not unlike some young Beginners in Geometry, who before they are well acquainted with the Elements of that Science, presently set upon the Quadrature of the Circle, the Trisection of an Angle by the means of right Lines and Circles, the Duplication of the Cube, and the like: so our Theorists, not having laid a good Foundation, raise up but a weak Superstructure; whence it is not to be wondered, if their great Building immediately sinks, not without the disgrace of the Architects. But those who philosophize aright, ought to take another course, and proceed in a quite different Method; and tho' they do not pretend to form a World, an Earth or a Planet,

Planet, yet they may be able to lay down a sure Foundation, and firm Principles of the Mechanical Philosophy, and explain the Phenomena, which are thence to be deduced.



LECTURE II.

Of the Solidity and Extension of Bodies.

WE shall not here give a Definition of Body, taken from its intimate Nature or Essence, wherewith we are not perfectly acquainted, and perhaps never shall be: but according to a Rule laid down in the former Lecture, we shall define it by some of its Properties, that distinguish it from every other Being whatever. We therefore say, That a Body is, *what is extended, solid, and capable of Motion.*

THERE is no one, I suppose, so slow of Apprehension, as not readily to perceive, that every finite Body has Bounds or Terminations, which Terminations we call Superficies, and of these some one is distant from its Opposite; and again this Superficies (since it is not infinite) has Extremes, which we call Lines, and betwixt which there must of necessity be some distance. And moreover there are Terminations of these Lines, which we name Points, betwixt which also there is an Interval. From all these Distances taken together, we acquire a distinct Idea of Extension into its triple Dimension. For the Distance betwixt two opposite Surfaces of the same Body, is called its Thickness or Depth; the Distance betwixt two opposite Lines of the same Superficies, is said to be its Breadth; and the Distance betwixt both the

the Extremes of a Line, is named its Length. There is no Body that does not partake of this triple Dimension, and however small we suppose it to be, it is necessary that it should have Length, Breadth, and Thickness: and that which is in a Body, and is destitute of all these, is not a Body, but a Point; neither is it a Magnitude, but the Beginning or Termination of a Magnitude.

SOLIDITY is that Property of a Body, whereby it resists all other Bodies that press it on every side; and whereby, whilst it possesses any place, it hinders all other Bodies from entering into that place, tho' they are forced against it ever so violently. So, for example, if a Body is held betwixt one's hands, tho' it is pressed with ever so great a Force, it prevents one's hands from coming to a mutual Contact.

THIS is that Property, which most of the *Peripateticks* are wont to call Impenetrability; that is, whereby two Bodies are not suffered to be in the same place at once, or to penetrate each other: tho' I rather chuse, with an illustrious Philosopher of our Age, to call it Solidity. This Property seems to be so essential to all Bodies, that there is no other Being in Nature to which it can belong: for tho' there are other sorts of Magnitudes, corporeal Magnitude alone admits of Solidity; all other Quantities, or indeed such as are no Quantities, as Points, can penetrate, unite, and be in the same place. For if two Spheres meet each other, in the place of Concourse there will be a Point of one united with a Point of the other, or they will agree, or will be in the very same Point of Space: So if two Cubes are equal, the one may be so placed upon the other, that two of their square Surfaces may agree; namely, the Sides of one of the Squares may coincide with the Sides of the other Square; and the Angles of one be united with the Angles of the other,
whence

whence these Quantities mutually penetrate each other, and are in the same place, which is impossible to happen in Bodies.

Y O U may easily perceive, that we use the word *Solidity* in a sense very different from that of the Geometers, who suppose that Solids may mutually penetrate each other. As, for Example, when *Euclid*, in the eleventh Book of his Elements, demonstrates the Equality of two Parallelepipedons, placed upon the same Base, and betwixt the same parallel Planes; it follows, that two different Parallelepipedons so placed must necessarily penetrate each other: whence it is manifest, the Geometers suppose their Solids penetrable. The Geometers therefore take the word *Solidity* in a sense different from that of the Philosophers, neither do they oppose their Solids to penetrable Magnitudes, but to Planes, or Surfaces, to plane Angles, and Lines; for every thing is reckoned solid by them, that has three Dimensions.

B U T the Solidity of Bodies is of another sort, which as we have said that it belongs to Bodies alone, so it is common to every Species of Bodies, whether they are fluid, or hard and fix'd, or easily moved and yielding to any Force, or very heavy, or whether they have but little Weight, or are absolutely light, if such Bodies can be in Nature; for a Drop of Water or a Particle of Air remaining fixed betwixt any two Bodies, does not less hinder the Contact of those two Bodies, than the hardest Metal or a Diamond would do.

L A S T L Y, By this Property, Body is distinguished from another kind of Extension, which we conceive to be penetrable, which we call Space, and wherein we behold all Bodies to be placed and moved, at the same time regarding that as immoveable.

T H E *Cartesians*, who define Body by its Nature, which they make to consist in Extension alone, allow
of

of no Space or Extension that is not corporeal; but since we have, or at least imagine we have, an Idea of Space distinct from the Idea of Body; they certainly offend against the Laws of good Method, who place the Nature or intimate Essence of Body in any of its Attributes, which, we are not sure, do only belong to it.

BUT the *Cartesians* say, that the Nature of Body cannot consist in any other of its Attributes, since neither Hardness, or Colour, or Weight, or Figure, or Taste, or any of its sensible Qualities, can constitute its Essence. For all these Attributes may be taken from the Body, and yet the Nature of Body will remain; but take away Extension, and the corporeal Being is immediately destroyed: therefore it is necessary that the Nature of Body should be placed in Extension alone.

THIS is the Argument of *Cartes* himself, tho' it is unworthy a Philosopher; for nothing can thence be deduced, unless this, that those sensible Qualities that he mentions, are not of the Essence of Body, and that Extension is an Attribute necessary and essential to it. But what then? Cannot the same universal Attribute agree to two different Species of things? Is it necessary that all things that have the same Attribute, must have the same Nature, and Essence? If this be so, there will be no Distinction, no Diversity in things. Tho' therefore Space and Body have one and the same essential Attribute common to them both, yet they are very different things; and there are other essential Attributes, peculiar to each, whereby they are sufficiently distinguished.

IN the first place, the above-described Solidity is proper to Bodies only, and so essential to all of them, that you cannot so much as separate it from them in your Imagination, but at the very same time you destroy that very Idea which you had formed of
Body:

Body; wherefore if the Essence and intimate Nature of Body is to be placed in some one Attribute, Solidity certainly has a much better pretence to be that Attribute than Extension; especially when there seems to be a Being different from Body, which we call Space, to which likewise Extension appertains: at least, the contrary does not yet appear.

BESIDES that we have the Idea of this Space altogether distinct from the Idea of Body, each of them seems to be endued with Attributes, not only distinct and proper to themselves, but so contrary, that it is impossible to imagine they could be inherent in the same Subject. For we conceive Body as solid or impenetrable, divisible and capable of Motion, whose Parts may be easily disunited, separated and removed from one another to any distance whatever: one Body may oppose another Body that is moving, it may stop, or at least diminish its Motion; a Body likewise may communicate its Motion to another Body that is at rest, or moves with a less force towards the same or opposite Parts, and carry it along with it.

ON the contrary, we conceive Space to be that, wherein all Bodies are placed, or, to speak with the Schools, have their *Ubi*; that it is altogether penetrable, receiving all Bodies into itself, and refusing an Ingress to nothing whatsoever; that it is immoveably fixed, capable of no Action, Form or Quality; whose Parts it is impossible to separate from each other, by any Force however great; but the Space itself remaining immoveable, receives the Successions of things in motion, determines the Velocities of their Motions, and measures the Distances of the things themselves. These so disagreeing and repugnant Attributes of Space and Body, it is impossible should belong to the same Subject.

THE *Cartesians* perhaps may reply, That this Idea, which we have here given of Space distinct from Body, is altogether imaginary and chimerical, the like to which cannot, they say, be made to exist in the Nature of things, by any Power whatsoever. But we are ready to demonstrate against the *Cartesians*, that in truth there is given a Space distinct from Body; or, in other Words, that Space and Body are not the same things: but we shall first of all observe, that we do not now intend to prove the real Existence of Space void of all Body, for that we shall do in another Lecture; we shall at present be content, only to shew the Possibility of it.

LET us suppose then any Vessel, and let it at first be filled with Air; then let the Air contained in this Vessel be exhausted, or, if you will, annihilated by a Divine Power, and let all other Bodies whatever be hindered from entering into its Place: I would now ask, whether in this Case there is not given a Space void of all Bodies? All that Body which was within the Vessel is destroyed, the Ingress of any other Body is prevented, and the Form of the Vessel is supposed to be preserved; it seems therefore to be necessary that a Vacuum, or a Space not replete with Body, is given. The *Cartesians* may answer, That on these Suppositions the Sides of the Vessel must fall in, and necessarily approach one another. But since, according to the *Cartesians* themselves, no Body can move of itself; and since, by Hypothesis, there is no other Body that forces the Sides of the Vessel towards one another; it therefore follows, that the Sides will not approach one another. They will say, perhaps, that the Air diffused every way, and pressing on all hands the Sides of the Vessel, will cause that Motion. But since the Pressure of the Air is of a finite Force, the Strength of the Vessel may be sufficient to balance that Pressure, and
confe-

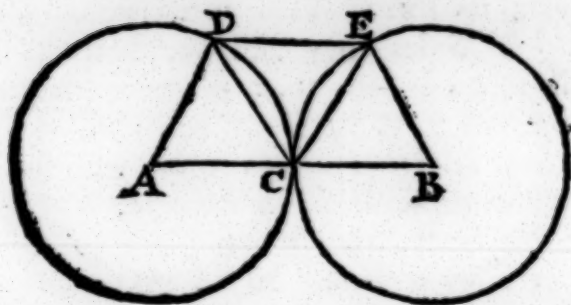
consequently its form will be preserved. But let us grant that its Sides may fall in, I demand, what Body will succeed into their place? (They will answer,) The Air. What Body will possess the Space left by that Air? (Perhaps they will say,) More Air? But we must stop somewhere, and at length come to some Body, into whose place no other Body will succeed; for it is absurd to go on thus *ad infinitum*: therefore in that case, it is necessary to suppose a *Vacuum*.

BUT we shall also shew, by an invincible Demonstration taken from Geometry, at least the possible Existence of a Space void of all Body: towards which Proof, we shall premise the two following Assertions, or Axioms, whose Evidence no Philosopher will call in question. The first is, That no Body or Portion of Matter stands in need of the Existence of another Body, in order to its own Existence. As, for example, a Sphere may exist, whether any other Body exists or not: this evidently follows from the Nature of Substance. Secondly, Any Body, especially if it be hard, can preserve its Figure, if there are no external Bodies or no Agents, which endeavour to induce an Alteration in it. Certainly it must be confessed, that God can preserve any Body in the same State and Position; and whatever may happen externally, he can notwithstanding continue the Figure of that Body immutable.

SINCE therefore one or two Spheres may exist, without the Existence of any other Bodies; let us suppose all other Bodies, except two Spheres, to be annihilated by God; or rather let us suppose all the Matter in the Universe to be amassed into two Spheres, which may be represented by two Circles, whose Centers let be A and B: and since no other Body is supposed to exist, those spherical Bodies may preserve their Figure, there being no other external

C

Body



Body either to destroy or alter it. These two Spheres then are either contiguous or disunited; if they are disjoined, there will be an intermediate Space replete with no Body, and therefore all Space is not Body: But if these Spheres touch one another, it is necessary that they touch one another in one Point only, by the Elements of Geometry: therefore betwixt the other Points of the Spheres there is some Distance, that is, some Space will lie between. For out of the Point of Contact let there be taken any two Points, as D and E; if betwixt these Points there is no Space, that is, no Distance, these Spheres will touch one another in these Points, which is impossible.

AGAIN, That there is a Space void of all Body, may be thus shewn. Let us suppose two Spheres, in which all the Matter of the Universe is accumulated, and that these two Spheres are equal; in both which let be accommodated the right Lines CD, CE, each equal to the Semidiameter of either Sphere, join DE; this right Line will be equal to the Semidiameter of one of the Spheres. For, draw AD, BE, and because in the equilateral Triangles ACD, BCE, the Angles ACD, BCE, are each the third part of two right Angles, the Angle DCE will be equal to the third part of two right Angles; for all the Angles at the Point C, make two right ones: whence since DC, CE, are equal, the Angles CDE and CED will be likewise equal; but taken together, they make two Thirds of two right Angles: wherefore each is one third

third part of two right Angles, whence the Triangle DCE is equi-angular; and consequently, DE will be equal to the Semidiameter of each Sphere, nor in this case can it be more or less. In like manner, betwixt any other Points of the Spheres out of that of Contact C, there will be some Distance, having a determinate Ratio to the Diameter of the Spheres; and therefore there will be betwixt these Spheres a certain and determinate Space, not replete with Matter: but into that Space may be admitted a Body, whose Dimensions agree with these Distances; but what has greater Dimensions, cannot by any Power whatever be placed within that Space. Whence, since these Properties demonstratively agree with the Space here spoken of, and that Space may really exist, tho' no one should think at all, it evidently follows against the *Cartesians*, that the Idea we have of Space, is not chimerical or imaginary; for what is chimerical, can have no existence out of the Intellect.

WE conclude therefore, that there is in reality a Space distinct from all Body; which is as a universal Receptacle, wherein all Bodies are contained and moved. But what is the Nature of this Space, whether it is any thing positive, actually extended in itself, and endued with real Dimensions, or whether its Extension arises from the Relation of Bodies existing in it, so that it may be a mere Capacity, Possibility, or Interponibility, as some love to express themselves, and to be reckoned in the same Class of Being with Mobility and Contiguity; or whether this our Space is the divine Immensity itself, which is through all and in all, or whether it is created or uncreated, finite or infinite, dependent or independent on God; these things we do not here enquire into, but leave them to be disputed by the Metaphysicians. It serves our turn to be able to explain some of its Properties, and to establish and demonstrate its Dis-

stinction and Nature to be different from that of Body: They who would have more, may consult the Philosophers.



LECTURE III.

Of the Divisibility of Magnitude.

ALTHOUGH it may be demonstrated by many Arguments, that Space is really distinct from Body, and we have already produced some, that seem to be unanswerable; yet both these things agree in this, that Extension is a universal Attribute, necessarily and essentially belonging to both of them. Before therefore we proceed any farther, it may not be foreign to the purpose to explain some general Affection of Extension; as for example, its Divisibility.

THIS Property of Extension appertains and necessarily adheres to all Species of Magnitude, as well to Lines as Surfaces, and as well to Space as to Body. By Divisibility we would not here be understood to mean an actual Separation of Parts from one another, which supposes Motion, which indeed the Nature of Space does not admit, nor do the Demonstrations borrowed from Geometry prove such a Separation; but the Divisibility that we here endeavour to evince, is only the Resolution of any Magnitude into its Parts, or their Distinction and Assignment. As, for example, when *Euclid*, in the ninth Proposition of his third Book, teaches how to cut a rectilinear Angle into two equal Parts, he does not in that Method undertake to shew, how one of the equal Parts being separated from the other, re-
cedes

cedes and is placed from it at a given Distance; but only delivers a Method, whereby a Line may be drawn, dividing the Angle in such a manner into two other Angles, that the Angle which lies on one side of this Line, shall be equal to the Angle that lies on the other side the same Line. So likewise when, in the following Proposition, he teaches how to bisect any right Line, he only shews how to assign a middle Point, dividing the given right Line into two equal Parts, which Point is the common Termination of both the Parts; namely, where one of the equal Parts ends, and the other begins. This Resolution of Magnitude into its Parts, is so intimate and essential to it, as that which has no Parts, as, for instance, a Point, is not said to be a Magnitude, but the Beginning or End of Magnitude: nor can any Magnitude be produced by any Number of Points, tho' infinite; for every Magnitude is not compounded of Points, but Parts, that is, other Magnitudes of the same kind, whereof every one is constituted of other Parts, and each of these is still made up of others, and so on *in infinitum*: nor can we ever arrive at a Magnitude so small, but it may be yet farther divided into Parts; nor is there given, in any Species of Magnitude, an absolute *Minimum*, but whatever is divided, is still farther divisible into Parts. This constant farther Resolution of Matter into Parts, is by the Philosophers called its *Divisibility in infinitum*; and that very truly, since there cannot be assigned any Quantity of Matter so minute, and any finite Number so great, but that the Number of Parts composing that Magnitude, that is, into which it may be resolved, shall be greater than that Number, how large soever it be: *for we call that Infinite, which exceeds any Finite.*

BUT because this infinite Divisibility of Matter can be demonstrated by Arguments taken from Geo-

metry, and since there are now-a-days some Philosophers who attempt to banish Geometry out of Physicks, by reason they are ignorant of that divine Science; and as these Gentlemen would be reckoned amongst the most Learned, they leave no means untried, whereby, tho' in vain, they may overturn the Force of these Demonstrations: It will be therefore necessary, before we produce our Geometrical Arguments, to establish their Strength, and to answer some Objections.

As amongst the Philosophers of this Class, the famous *John Baptist du Hamel*, the Author of the *Burgundian Philosophy*, is of the greatest Eminence; we shall produce his Opinion on this Subject. He says then, that Geometrical Hypotheses are neither true nor possible, since neither Points, nor Lines, nor Surfaces, as the Geometers conceive them, do truly exist in the Nature of Things; and therefore that the Demonstrations that are produced from these, cannot be applied to things actually existing, when none of these exist any where but in our Ideas. He desires therefore the Geometers to keep their Demonstrations to themselves, and not to make use of them in Philosophy, because, according to him, they spread over this Science not Light, but Darknes. I admire at the Unskilfulness of this otherwise most Learned Person, in this Affair: he might certainly with the same Justice take away all Physical Suppositions whatsoever, since Geometrical Hypotheses are equally certain and equally possible and real, as are those, which he calls Physical. Certainly if Body exists, there must of necessity exist real Points, real Lines, and real Surfaces, even such as are conceived by the Geometers; as we can easily make appear. For if Body be given, that, since it is not infinite, has its Terminations; but the Terminations of Body are Surfaces, and those Terminations have no Depth: for if they had, they

they would thereby be Bodies, which Bodies would have still other Terminations, which would be Surfaces, and therefore there would be a Superficies of a Superficies. Either then this Superficies is destitute of all Depth, or not: if the first, we have what we require; if the latter, we come again to another Superficies, and so we should proceed *in infinitum*, which is absurd. Wherefore we must conclude, that those Terminations are deprived of all Depth, and are therefore true Surfaces, and as they are conceived by the Geometers without any Depth, or such as have only Length and Breadth to constitute their Essence.

AGAIN, Since this Surface is not infinite, it is likewise bounded by its Terminations; but those Terminations are called Lines, which have really no Breadth: for otherwise they would be Surfaces, and would have also their Terminations, which we ought to conceive at least destitute of all Breadth; for, as we said before, there cannot be given a Progression *in infinitum*; whence there are really given Lines, which are only extended in Length, without any Breadth. After the same manner, Lines also have their Terminations, which are called Points, to which belong neither Length, nor Breadth, nor Depth. Wherefore if Body may be supposed to exist, it necessarily follows, that Geometrical Surfaces, Lines, and Points, may be said not only as possible to be, but also to be actually existing.

BUT it will be answered, That these Points, Lines, and Surfaces are not material. What then? Who ever asserted that a Mathematical Point was Matter? Who ever fancied a material Superficies? If it was material, it would have its Superficies or Termination: but the Superficies of a Superficies, who ever imagined? However, tho' neither Surfaces, nor Lines, nor Points are real Matter; yet

they exist or may exist in it, as its Modes, Terminations, or Accidents: just after the same manner, as Figure is not Body itself, but only its Affection, whereby it is contained under given Terminations, and this has real Properties wholly distinct from those of Body.

AND again our ungeometrical Philosophers may object, That there is not in Nature a perfectly plane Superficies, a Body perfectly spherical, such as the Geometers feign, nor any Curve perfectly circular. But how come they to know all this? Have they seen all the Bodies that are in the Universe, and view'd them thro' a Microscope? Perhaps they will say, that a Superficies cannot be plane or spherical, because in these Figures there is a Contradiction and an Impossibility. But we desire they would be pleased to make out this Contradiction: All Bodies of necessity are terminated by some Figure, plane or spherical Surfaces are the most simple and easiest to be conceived of any; where then lies the Repugnancy, that it should be impossible for a Body to be contained under these Surfaces? I believe there is no one who is acquainted with the Elements of Geometry, but perceives more the Nature and Properties of these Figures, and knows more of their Affections, than all the Philosophers of this sort put together: but none of those ever found any Repugnancy in these Figures; no Geometer ever suspected these Contradictions in the Nature of Figures: on the contrary, so many beautiful Properties of those Figures discovered and demonstrated by the Geometers, evince their Possibility, for of an impossible thing there can be no real Property, no Demonstration. It remains therefore, that they acknowledge these Figures as possible; and if they are possible, it is in the power of God to form out of Matter, Bodies having such Surfaces. Let us therefore suppose

two Bodies, whereof one is terminated by Planes, the other by a spherical Superficies; if therefore the spherical Superficies is placed on the Plane, it will touch it; but it will touch it only in one, and that an indivisible Point, or in a Point that has no Parts, by *Cor. Prop. 2. El. 3.* and therefore in that case there will be given a true Point. But farther, let us suppose this spherical Body to be moved on the plane Superficies, or to be carried along without any Rotation about its Axis, infomuch that the Point touching the plane Superficies may be always found in the same Plane; then the Tract which that Point describes by its motion, will be a Mathematical Line, without any Breadth: and if the Distance betwixt any two Points in that Plane is the shortest, there will be generated by that Motion a right Line; but if otherwise, either a Curve, or one compounded of right Lines, or partly of these, and partly of those, will be described. Points therefore, Lines, and Surfaces, even such as the Geometers conceive or feign, are possible; which was to be shewn. And their Possibility may be demonstrated after innumerable other ways; but we are weary of dwelling on these Trifles. We shall only farther observe, that the Distance betwixt any two Points of two Bodies, will be their given and determinate Distance: As, for example, the determinate Distance betwixt the Centers of the Sun and a fixed Star, is that which is measured by a right Line lying betwixt those two Points; which, of all the Lines that can be drawn from one Point to the other, will be the shortest, and passed over in the least time with a given Velocity: this Distance, I say, will remain the same, whatever shall happen to be the Figure of the intermediate Body, whether it is bounded by Planes, or contained under spherical Surfaces, or lastly if there should be no intermediate Body, and nothing but Space lie betwixt; the

the Line will remain the same in Magnitude and Position, as long as the Centers of the Bodies remain unmoved.

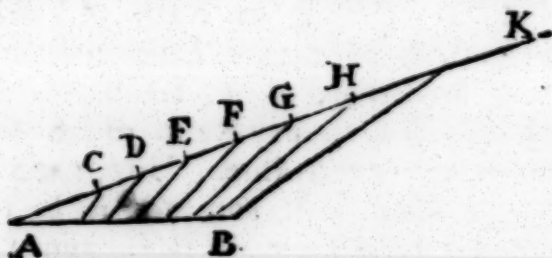
HAVING now settled these Principles, we return to our Purpose; which was to demonstrate that all Extension, whether corporeal or incorporeal, was divisible *in infinitum*, or had an infinite Number of Parts; which we shall endeavour to prove by many invincible Arguments. Of which, this shall be the first: Let AB represent a right Line, I say it is divisible into Parts exceeding any finite Number whatever.



THROUGH A let be drawn any right Line AC, and parallel to it let be drawn through B the right Line BD, and in AC let there be taken any Point, as C: if therefore the right Line AB is not divisible into an infinite Number of Parts, let it be divisible only into a finite Number of Parts; and let that Number, for example, be fix. In the Line BD on the side opposite to C, let there be taken any Number of Points exceeding fix; for Example, the Points E, F, G, H, I, K, L, and let there be drawn by the first Postulate of *Euclid*, CE, CF, CG, CH, CI, CK, CL. These thus drawn, divide the right Line AB into as many Parts as there are right Lines; for if they do not, then some of the right Lines intersect AB in one and the same Point: but all of them intersect one another in the common Point C, whence
some

some two right Lines will cut one another twice, or will have the same common Segment; both which is contrary to an Axiom in the *Elements*. AB is therefore divided into as many different Parts, as there are right Lines; but there are as many right Lines, as there were Points taken in the right Line BD: wherefore since there were taken more Points than six, the right Line AB is divisible into more Parts than six. After the same manner, how great soever the Number assumed shall be, it may be shewn that the Line AB is divisible into a Number of Parts greater than that Number; namely, by taking in the right Line BD a greater Number of Points, (which may be easily done, since no finite Number is so great, but a greater may be assumed, and that in any given Ratio of a greater Inequality) and by drawing right Lines from the Point C to the Points taken in the right Line BD: for these right Lines will divide the right Line AB into as many Parts, as there are right Lines, and therefore into more Parts than the Number first assumed (how great soever it was) contains Units; and consequently the right Line AB is divisible into more Parts than can be expressed by any finite Number, and therefore it is divisible *in infinitum*. Q. E. D.

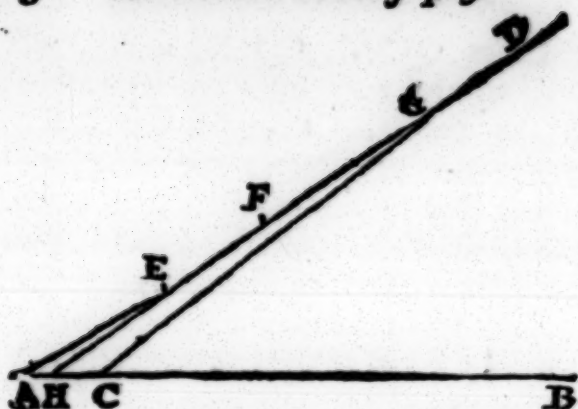
THE second Argument. Let AB represent any right Line, I say it is divisible into an infinite



Number of Parts; for if it is not divisible into an infinite Number of Parts, let it be divisible into a
I
finite

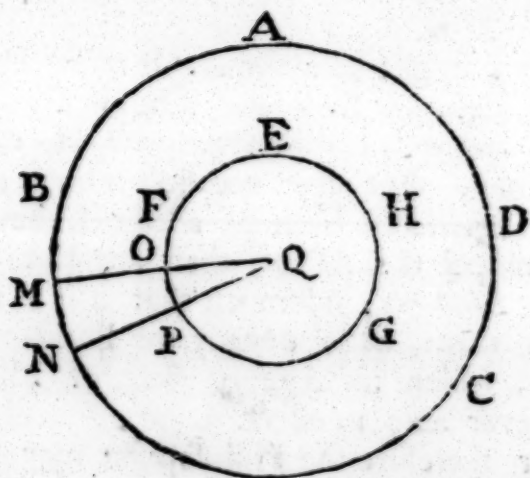
finite Number of Parts; and let that Number be, for example, five. Let any right Line A K be drawn, making any Angle with A B, and in it, produced if necessary, let there be taken as many Points as you please above five, which let be C, D, E, F, G, H, K; join K B, and through the Points C, D, E, F, G, H, let right Lines be drawn parallel to K B: these will necessarily divide the right Line A B into as many Parts as there are right Lines; for if they do not, more right Lines must concur in one and the same Point: but they cannot concur, since they are parallel; wherefore each right Line will intersect the right Line A B in a different Point, and all will divide the right Line A B into as many Parts, as there were right Lines drawn parallel. But there were more drawn than five, therefore the right Line A B will be divided into more Parts than five. And the same may be affirmed of any other Number. Wherefore no Number is so great, but the Number of Parts the right Line is divisible into, is still greater; and consequently the right Line A B is divisible *in infinitum*.

THIRDLY, If Quantity is not divisible *in infinitum*, it must be divisible into Parts, that are not farther divisible; but there is no Part that cannot be still farther divided, because there can be given no Quantity so small, but there may be still taken a smaller, and that in any given Ratio of lesser Inequality. For let A B be a right Line, and let A C be an exceeding small Part of it; I say, there may be a Line less than A C in any Ratio of less Inequality, as, for example, one to three. From the Point A draw any right Line A D, and in it let be taken the equal right Lines A E, E F, F G; join G C, and through E draw E H parallel to G C, the right Line A H will be a third Part of A C: The Demonstration thereof is manifest from the *ninth Proposition of the sixth*



sixth Book of the Elements. And therefore the right Line AC will not be the least that can be taken. The same may be demonstrated of any right Line whatsoever; consequently there is not in Nature an absolute *Minimum*.

AGAIN, If Quantity was compos'd of Indivisibles, many Absurdities would thence follow: for let there be, for example, two concentrick Circles, ABCD, EFGH, and let the Circumference of the



greater be divided into its indivisible Parts, and let be drawn from the Center Q to each of these Parts, the

the right Lines QOM, QPN, which will divide both the Circumferences into an equal Number of Parts, but the greater Circumference ABCD was divided into its smallest Parts; and therefore the less Circumference EFG will consist of as many Indivisibles, or smallest Parts possible, as the greater Circumference ABC: And consequently since an Indivisible is equal to an Indivisible, the Circumference EFGH will be equal to the Circumference ABCD, a less to a greater, which is absurd.

LASTLY, From this Composition of Quantity of Indivisibles, there can be no incommensurate Magnitudes; which is contrary to what the Geometers frequently demonstrate. For if all Magnitude consisted of Indivisibles, an Indivisible would be an adequate and common Measure of all Magnitudes of the same kind; for it would be exactly contained some number of times in all, and therefore all Magnitudes would have a common Measure, and the Side of a Square would be commensurate to its Diagonal; contrary to *the last Proposition of the tenth Book of Euclid's Elements*.

THERE might be innumerable other Demonstrations produced, to shew the infinite Divisibility of Quantity, and entirely to overthrow the Hypothesis of Indivisibles. But what occasion is there for more? since the Arguments hitherto alledged, have not less force to compel the Assent, than any Demonstration in *Euclid's Elements*; insomuch that it is as impossible to weaken them, as to destroy the Fundamentals of Geometry, which no Age or Sect of Philosophers has been ever able to effect.

THAT therefore the Philosophers may avoid the Force of these Arguments, they distinguish betwixt a Mathematical and a Physical Body. Being compelled by the Force of Demonstration, they readily allow a Mathematical Body may be divisible in *infinitum*;

nitum; but they deny that a Physical Body can be always resolved into still farther divisible Parts. But what, I would know, is a Mathematical Body, but something extended into a triple Dimension? Does not Divisibility belong to a Mathematical Body, by reason it is extended? But a Physical Body is extended after the same manner: wherefore since Divisibility depends on the Nature and Essence of Extension itself, and owes to it its Origin, it is necessary that it must agree to all Extensions, whether Physical or Mathematical. For, to use a Logical Expression, whatever is predicated of any *Genus*, is predicated of all the *Species* contained under that *Genus*.

T H E R E is yet another Distinction amongst the Philosophers, not unlike the former; whereby they own that every Body is mathematically divisible *in infinitum*, but they deny that it is physically so. If these Words have any Meaning, it is certainly this: They acknowledge that a Body is mathematically, that is, really and demonstratively, divisible *in infinitum*, but they deny it to be so physically, or according to their false *Hypothesis*: and so they have a Distinction, against which nothing can be replied.

B U T because the Philosophers, against whom we dispute, are not acquainted enough with Geometrical Demonstrations, and therefore do not easily perceive their Evidence; before we end this Lecture, we shall produce one Physical Argument taken from Motion, for the infinite Divisibility of Quantity: namely, If Quantity consisted of Indivisibles, it would follow, that all Motion would be equally swift, nor would a slow Snail pass over a less Space in the same Time than the swift-footed *Achilles*. For let us suppose *Achilles* to run very swiftly, and the Snail to creep sluggishly along; if Extension consisted of Indivisibles, the Snail could not in any given
Time

Time pass over less Space than *Achilles*: for if in a Moment's Time *Achilles* passes over an indivisible Space, the Snail cannot in the same Moment of Time pass over less Space; by reason, from the Hypothesis, there cannot be a less. For one Indivisible cannot be less than another, therefore it will pass over an equal Space. The same may be said of any other Moment of Time: Therefore the Spaces passed over by them both will be equal; and consequently the swift-footed *Achilles* cannot pass over more Space than the slowest Snail: which is absurd. Other Absurdities of the like sort, may be deduced from the same Hypothesis of Indivisibles; but what we have already said, is sufficient.



LECTURE IV.

Wherein the Objections usually brought against the Divisibility of Matter, are answered.

WE have hitherto produced such Arguments, as by their help we have sufficiently demonstrated the continual Division of Matter into an infinite Number of Parts: it now remains, that we answer to the Objections or Cavils of the Philosophers. For there are not a few of them, who labouring under I know not what Obscurity of Ideas, and not clearly enough perceiving the Evidence of the Arguments we have here made use of, produce Arguments of their own against a thing so manifestly true, and to which they are also pleased to give the Title of Demonstrations. But as I have
perused

perused many of their Books, I never lit on any thing in all their Writings upon this head, that had so much as the very Appearance of Reason: they were so far from coming up to Demonstrations, that no Geometer, however clear-sighted he was, could ever perceive in them the least Shadow of a Demonstration. I acknowledge there is something in the Nature of Infinites, that seems not to be adequately comprehended by the human Intellect; and therefore it is no wonder, if some things follow from it, which the Mind of Man, involved in thick Darkness, is not able to conceive: and especially in our present Question, there are many things that may seem as Paradoxes, and incredible, to such Philosophers, who are less conversant in these Matters. However, nothing thence follows, that implies either a^d Contradiction, or is repugnant to any Axiom or Demonstration. But let us consider the Cavils produced by the Atomical Philosophers. The first is that of *Epicurus*: If Quantity was divisible *in infinitum*, it would contain an infinite Number of Parts, and so a Finite would contain an Infinite, which is absurd. But I desire they would explain their Terms, and tell us what they mean by these words, *An Infinite cannot be contained by a Finite*. If they say an infinite Magnitude cannot be contained in a finite Magnitude, I acknowledge it; but the contrary to this does not follow from our Doctrine, nor can they thence ever deduce it by a necessary Consequence. If they say that an infinite Number of Parts, and those infinitely small, cannot be contained in a finite Magnitude, this is the very thing they ought to make out. They would not, I suppose, have us believe their Assertions without any Proof; nor would they place amongst the Axioms, as a self-evident Proposition, such a one, as we have demonstrated by so many strong Reasons the contrary to be true. They may therefore urge, that an infi-

D

nite

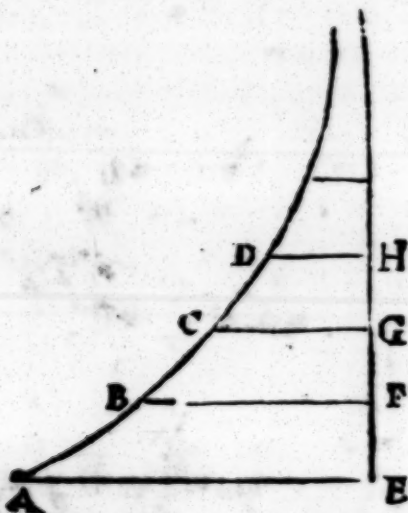
nite Number of Parts must compose an infinite Magnitude: but this is again to beg the question; for that is the thing itself about which we dispute, *viz.* Whether a finite Magnitude can have an infinite Number of Parts? It is certain, whatever Number of Parts it has, whether finite or infinite, they are equal to their Whole; for as ten tenth Parts of an Unit make an Unit, an hundred hundredth Parts of an Unit taken together do likewise compose an Unit, and a thousand thousandth Parts collected in one Sum cannot be greater than the Whole; so likewise the infinite infinitesimal Parts of any Magnitude are equal to that Magnitude. Or thus: Let the right Line A B be divided into an hundred Parts; all these taken

$$\begin{array}{ccc} A & & B \\ \hline C - & C : AB :: 1 : N \end{array}$$

together will be equal to A B. And after the same manner, if the right Line A B be supposed to be divided into a thousand Parts, these thousand Parts taken together will compose a Magnitude neither greater or less than the right Line A B. Or again, if the right Line A B be divided into a million of Parts, these taken together will be equal to A B the Whole. And universally, if there be taken two Magnitudes A B and C, so that C shall have the same Ratio to A B, as a Unit has to any Number N, the Quantity C multiplied by the Number N will be equal to A B. For since the Quantities C, A B, Unity and the Number N are Proportionals, the Extremes and the Means multiplied into each other will be equal; but since A B multiplied by a Unit is equal to A B, (for a Unit neither increases by Multiplication or diminishes by Division) the Quantity C multiplied by the Number N, will be equal to A B: therefore however great or small the Number N is,

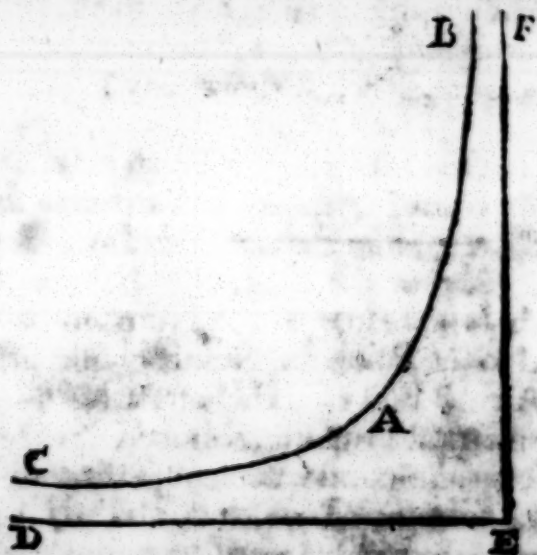
is, this multiplying the Quantity C will always make a Product equal to A B, if the Quantity C be so taken, that it has the same Proportion to A B, as Unity has to the Number N. And therefore if N is an infinite Number, and C the infinitesimal Part of the right Line A B, that is, the Quantity C has the same Ratio to A B, as a Unit has to an infinite Number N; also the Quantity C multiplied by an infinite Number N, that is, taken an infinite number of times, will be equal to the Quantity A B, nor as it can be greater, neither can it be less. If therefore the Magnitude of the Parts is diminished in the same Ratio as their Number is increased, the Whole made up of all these Parts will remain the same: nor ought any Quantity to be measured from the Number of its Parts, but from their Number and Magnitude conjointly; so that if the Parts are infinitely small, it is necessary that their Number should be infinitely great, before they can exceed any given Quantity. But besides, there may be many Examples brought both from Arithmetick as well as Geometry, where, by the confession of our Adversaries themselves, the Number of Parts will be infinite, but the Magnitude composed of those Parts shall be finite. Let the first Example be a Series of Numbers decreasing in any proportion, which shall be equal to a finite Number, as, for example, $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \frac{1}{32}, \frac{1}{64}, \&c.$ the Sum of this Series continued *in infinitum* will be equal to a Unit; but since the Series is continued *in infinitum*, its Terms will be infinite in Number: wherefore in this case the Parts of a Quantity, that are infinite in Number, will make a finite Quantity. And in like manner the Sum of this Series, $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \&c.$ when continued *in infinitum*, will be equal to half an Unit, as may be demonstrated by Arithmetick. But no body will deny, that this Series, being continued *in infinitum*, has an infinite Num-

ber of Parts; wherefore there may be an infinite Number of Parts of a Quantity, which however shall not exceed one half of an Unit. And in Geometry it is known that there may be given a Space infinitely long, which however shall be exactly equal to a finite Space; and this in an infinite number of Examples has been demonstrated by the famous Geometers, *Torricellius*, *Wallis*, *Barrow*, and others; from whom we shall produce a few Instances. And first of all, let the Curve *ABCD* be of such a nature, that if there are taken in the Asymptote *EH* the equal right Lines *EF*, *FG*, *GH*, or the right Lines



EF, *EG*, *EH* are supposed in Arithmetical Proportion, and to the Points *E*, *F*, *G*, *H*, are applied the Ordinates *AE*, *BF*, *CG*, *DH*, which Ordinates let be in a Geometrical Proportion: the Curve *ABCD* is call'd the Logarithmetical Curve, and the endless Space contained betwixt the Asymptote and the Curve infinitely produced, will be equal to a finite Space, as is demonstrated by the Great Dr. *Barrow* in his *Geometrical Lectures*. Whence may be shewn the above-mentioned Property of Numbers decreasing in any Geo-

Geometrical Proportion. But that we may apply his to our present Purpose; nobody will deny that there are, in the boundless Space $HGFE\ ABCD$, which is infinitely long, an infinite Number of Parts, but the Geometers demonstrate all those Parts to be equal to a finite Space; wherefore there are some Parts of Space, tho' infinite in Number, yet they do not make an infinite, but a finite Space. After the same manner, in all Hyperbolas, except that of *Apollonius*, the Area contained betwixt the Curve and the Asymptote infinitely produced, will be perfectly quadrable, and equal to a finite Area; but in all these Areas there are an infinite Number of Parts, wherefore an infinite Number of Parts will be equal to a finite Quantity. Besides, in the *Apollonian* Hyperbola CAB , although the indeterminable Area contained betwixt the Curve AB and its Asymptote EF produced *in infinitum*, is an infinite Area, or



greater than any finite one; yet if that infinite Area revolves about its Asymptote, it will thereby generate a Solid, or Body truly infinitely long, which however will be equal to a finite Solid or Body; as

has been most elegantly demonstrated by *Torricellius*, who called this Solid, *Hyperbolicum acutum*: but in this Solid there are an infinite Number of Parts, since it is infinitely long; therefore the Parts of a Body infinite in Number, may compose a finite Body. We might produce innumerable other Examples of this, but we have perhaps bestowed too much time in answering this Objection.

SECONDLY, The Atomists object, If all Quantity is divisible *in infinitum*, any the least Magnitude will be equal to the greatest, since the least has as many Parts as the greatest. But what consequence, I pray, is this? Because a Yard may be divided into an hundred Parts, and a Foot may be likewise divided into the same number of Parts, therefore does it follow that a Foot is equal to a Yard? But there can be nothing found more alike, than this Way of Argumentation and the present Objection; which is grounded on a most false Hypothesis, which supposes that Magnitudes are to be measured by their number of Parts alone, without any regard to their Quantity.

THEY farther object, If a Foot may be divided into infinite equal Parts, and a Yard may be so divided, that every Part of the Yard may be equal to one of the Parts of the Foot, the Number of Parts in the Yard will be triple the Number of Parts in the Foot; whence since the Number of Parts in the Foot is infinite, the Number of Parts in the Yard will be triple that infinite Number, whence there will be given an Infinite triple of another Infinite. But whence do they learn that this is an Absurdity? Does it contradict any Axiom commonly received? Not at all, for there is no Axiom that supposes all Infinites equal. Nor is it contrary to the Nature of an Infinite, that there should be another Infinite still greater: for if there be given an Infinite, as, for
exam-

example, an infinite Line, there will be in it an infinite Number of Miles, more Furlongs, and still more Feet. And so in Space, which we suppose to be infinitely extended, if two parallel Lines be produced *in infinitum*, the Area contained under these two Lines will be really an infinite Area, for it will exceed every finite Area, or such as is bounded on all sides; it will therefore contain an infinite Number of Acres, more square Perch, and still more square Feet. Again, if betwixt these Lines be drawn another right Line parallel to either of them, this Line will divide the former Area into two other infinite Areas; which taken together, will be equal to the former infinite Area. It is not therefore repugnant to the Nature of Infinity, that one Infinite may exceed, be multiplied, or divided by another Infinite; these, I say, are no ways repugnant, but easily follow from the Nature of the Thing; and there is nobody that acknowledges an infinite Space, but at the same time he must be forced to own the Divisibility of that infinite Space into others that are also infinite.

ANOTHER Argument against the infinite Divisibility of Matter, is fetched from the Divine Omnipotence. God, they say, can resolve any Quantity into its infinitesimal Parts, and separate those Parts from one another: but if so, then may be given the ultimate part, and the Divisibility would be exhausted, therefore Quantity is not divisible *in infinitum*. I answer, without doubt God is able to do whatever is possible, or what is not repugnant to his immutable Nature; but since we have already demonstrated that there cannot be given any Particle of Matter, however small, which may not be still divided into other infinite Particles; it is thence manifest, that God cannot so divide Matter, as that there shall be given its ultimate Indivisible. For if

has been most elegantly demonstrated by *Torricellius*, who called this Solid, *Hyperbolicum acutum*: but in this Solid there are an infinite Number of Parts, since it is infinitely long; therefore the Parts of a Body infinite in Number, may compose a finite Body. We might produce innumerable other Examples of this, but we have perhaps bestowed too much time in answering this Objection.

SECONDLY, The Atomists object, If all Quantity is divisible *in infinitum*, any the least Magnitude will be equal to the greatest, since the least has as many Parts as the greatest. But what consequence, I pray, is this? Because a Yard may be divided into an hundred Parts, and a Foot may be likewise divided into the same number of Parts, therefore does it follow that a Foot is equal to a Yard? But there can be nothing found more alike, than this Way of Argumentation and the present Objection; which is grounded on a most false Hypothesis, which supposes that Magnitudes are to be measured by their number of Parts alone, without any regard to their Quantity.

THEY farther object, If a Foot may be divided into infinite equal Parts, and a Yard may be so divided, that every Part of the Yard may be equal to one of the Parts of the Foot, the Number of Parts in the Yard will be triple the Number of Parts in the Foot; whence since the Number of Parts in the Foot is infinite, the Number of Parts in the Yard will be triple that infinite Number, whence there will be given an Infinite triple of another Infinite. But whence do they learn that this is an Absurdity? Does it contradict any Axiom commonly received? Not at all, for there is no Axiom that supposes all Infinites equal. Nor is it contrary to the Nature of an Infinite, that there should be another Infinite still greater: for if there be given an Infinite, as, for exam-

example, an infinite Line, there will be in it an infinite Number of Miles, more Furlongs, and still more Feet. And so in Space, which we suppose to be infinitely extended, if two parallel Lines be produced *in infinitum*, the Area contained under these two Lines will be really an infinite Area, for it will exceed every finite Area, or such as is bounded on all sides; it will therefore contain an infinite Number of Acres, more square Perch, and still more square Feet. Again, if betwixt these Lines be drawn another right Line parallel to either of them, this Line will divide the former Area into two other infinite Areas; which taken together, will be equal to the former infinite Area. It is not therefore repugnant to the Nature of Infinity, that one Infinite may exceed, be multiplied, or divided by another Infinite; these, I say, are no ways repugnant, but easily follow from the Nature of the Thing; and there is nobody that acknowledges an infinite Space, but at the same time he must be forced to own the Divisibility of that infinite Space into others that are also infinite.

ANOTHER Argument against the infinite Divisibility of Matter, is fetched from the Divine Omnipotence. God, they say, can resolve any Quantity into its infinitesimal Parts, and separate those Parts from one another: but if so, then may be given the ultimate part, and the Divisibility would be exhausted, therefore Quantity is not divisible *in infinitum*. I answer, without doubt God is able to do whatever is possible, or what is not repugnant to his immutable Nature: but since we have already demonstrated that there cannot be given any Particle of Matter, however small, which may not be still divided into other infinite Particles; it is thence manifest, that God cannot so divide Matter, as that there shall be given its ultimate Indivisible. For if

the Divine Power extended thus far, then God could do something that involved a Contradiction, or that was repugnant to his immutable Essence. But they farther urge, If all Quantity is divisible *in infinitum*, and the Parts are actually in the Extension, there will be actually given a Part infinitely small, and consequently not farther divisible. In the first place, I answer, It may be denied with *Aristotle* that the Parts are actually in the Extension, and then their Argument falls to the ground, which they boasted of as an unanswerable Demonstration. Secondly, we grant them that these Parts are actually in the Extension; we grant that there are Parts infinitely small and indivisible; lastly, we grant the Argument: and yet nothing thence follows against the continual and infinite Divisibility of a Quantity that is not infinitely small. This is supposed indeed in the Argument, but without any Proof. Does it, because an infinite small Part of any Extension is not farther divisible, therefore follow, that a given Part, or one not infinitely small, is also not farther divisible? If any thing thence follows, it is, that all continued Quantity may be resolved into infinitely small Parts, and therefore it is infinitely divisible. But the third and true Answer is, by denying that the Parts in the Extension are so minute or small, that they cannot be farther divisible; and tho' there are given Parts infinitely small, or such as have the same proportion to their Whole, as a finite to an infinite Number, or a finite Space to an infinite one; yet we deny that these Parts are not farther divisible; but since they are extended, they will be also divisible, not only in two or three, or more Parts, but likewise every one may be divided *in infinitum*. The infinite Number of Parts of an infinitely small Quantity, are wont to be called by the Geometers, Infinitesimals of Infinitesimals, or Fluxions of Fluxions; and these are used
by

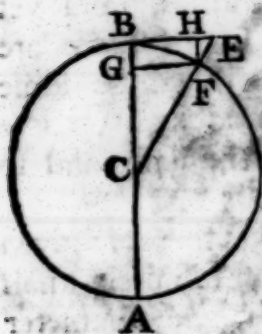
by them in the solving of many intricate Problems. Besides, there are given other Fluxions of these Fluxions, or Parts that are infinitely less than their Wholes; and again, there are other Parts of these Parts, and so on at pleasure. I do not deny, but, from the Weakness of the Human Understanding, this is very difficult to conceive: however, the Truth that is supported by such powerful Arguments is not to be deserted, especially since there are some things, which we do most certainly know, which yet are very difficultly received by our weak Understanding. We might produce many Arguments, but we shall only bring such as serve to illustrate our present Purpose; whereby we shall shew, that there are Quantities infinitely less than some Quantities, and yet also infinitely greater than others: so, if there are given infinitely small Quantities, there will be some Quantities infinitely less than these: and again, there may be others infinitely less than the last, and so always on *in infinitum*.

FIRST then, we shall thus prove, that there are given Quantities that are infinitely less than infinitely small Quantities. Let ABF be a Circle, whose Diameter is AB ; and let BF be an infinitely small Part of its Circumference, whose Chord therefore shall be also infinitely small, that is, the Chord BF shall have to any determinate Magnitude, as, for example, to AB the Diameter of the Circle, the same proportion, as any finite Magnitude has to an infinite one. Let fall from the Point F , on AB , the Perpendicular FG ; BG will be infinitely less than the right Line BF . For let be drawn AF , and the Angle AFB in the Semicircle will be a right one. And therefore in the



the Triangle AFB rectangular at F , from the Perpendicular FG being let fall on the Base AB , it will be, by the eighth of the sixth Book of *Euclid's* Elements, AB to BF as BF to BG ; but, by Hypothesis, AB is infinitely greater than BF , wherefore BF will be infinitely greater than BG . There may therefore be a Quantity, which, although it is infinitely less than a given Quantity, will be infinitely greater than another Quantity.

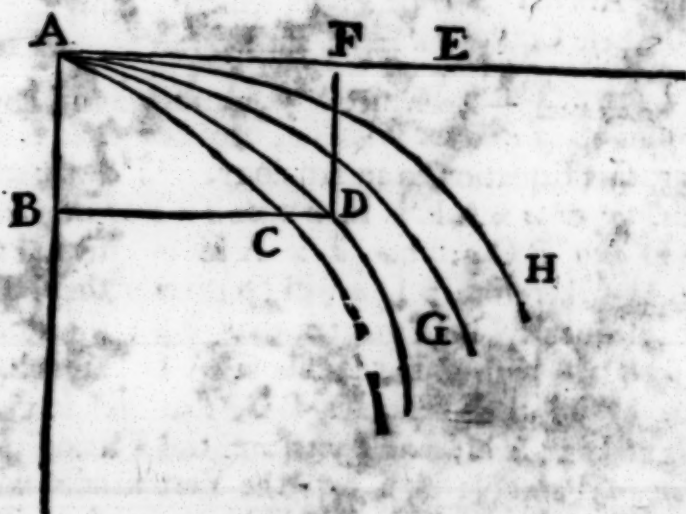
So likewise, it is known in the Circle, that as the Sine of any Arch is less, so the Tangent is greater than its correspondent Arch; and therefore the Tangent is greater than the Sine of the same Arch. Let therefore in the Cir-



cle, whose Center is C , and Diameter AB , BF be an infinitely small Arch, whose Tangent let be BE , right Sine GF , and versed one GB ; through F draw FH parallel to AB , then HE will be equal to the Difference of the right Sine FG and the Tangent BE , which, from what we have shewn, is not altogether nothing. Now in the Triangles CBE , FHE , equi-angular, by reason of the right Angles at H and B , and the common one E , it will be, by the fourth of the sixth of *Euclid*, CB to BE as FH to HE ; but by Hypothesis, CB is infinitely greater than BE , wherefore FH will be infinitely greater than HE : that is, in the present Case, BG the versed Sine of an infinitely small Arch, is infinitely greater than the Difference betwixt the right Sine and the Tangent of the same Arch. Since therefore CB is infinitely greater than BE ; and BE , as we have before demonstrated, is infinitely greater than BG ; and again, by what we have now shewn, BG is infinitely greater than HE ; the Proposition is manifest.

To

To the farther Illustration of this Doctrine, we shall produce another Example, borrowed from the Scholium of the first Section of the Great Sir *Isaac Newton's Principia*. Let the Curve $A C$ be the common Parabola, its Axis $A B$, and $A F$ a Tangent at the Vertex A . The Writers of Conicks demonstrate,



that as in the Circle, so in the Parabola, the Angle of Contact $F A C$ is infinitely less than any rectilinear Angle. To the Axis $A B$ and Vertex A , let there be supposed to be described a Parabola of another kind, as, for example, the Cubical, whose Ordinates increase in a subtriplicate Ratio of the Abscissas; the Angle of Contact $F A D$ will be infinitely less than the Angle of Contact of the Parabola $F A C$; or, which is the same thing, there can be described no *Apollonian* Parabolas, or no Circles, with how great soever a Parameter, that will pass betwixt the Cubical Parabola and its Tangent at the Vertex: which may be thus easily demonstrated. Let the Parameter of the *Apollonian* Parabola $A C$ be called a , the Parameter of the Cubical Parabola $A D$ let be b ; let there be taken in the Tangent the Point E in such a manner, that

that AE may be a third proportional to the right Lines a and b , that is, that $a \times AE = b^2$; through any Point F betwixt A and E let be drawn FD parallel to the Axis, meeting the Curve AD in D , and let BCD be drawn parallel to the Tangent, and let BD , an Ordinate to the Parabola AD , be called z ; but BC , an Ordinate to the Parabola AC , y ; and let the Abcissa AB be x : it will be from the Nature of these Curves $ax = y^2$, and $b^2x = z^2$, and therefore $\frac{y^2}{a} = x = \frac{z^2}{b^2}$; whence $b^2y^2 = az^2$, and so by re-

ducing this Equation to an Analogy, $b^2 : az :: z^2 : y^2$ that is, b^2 or $a \times AE$ is to az or $a \times BD$ or $a \times AF$ as BD^2 to BC^2 : but $a \times AE$ is greater than $a \times AF$, wherefore BD^2 will be greater than BC^2 , and consequently BD greater than BC ; the Point C therefore falls within the Parabola AD . The same is true of all the Ordinates BC , that are less than the right Line AE ; and therefore the Portion of the *Apollonian* Parabola AC , at the Vertex falls within the Cubical Parabola. The Demonstration is the same for any other *Apollonian* Parabola; consequently there can be drawn no Parabola, and therefore no Circle (which is always of the same degree of Curvity with some Parabola) betwixt the Cubical Parabola and its Tangent at the Vertex.

How much soever therefore the parabolical or circular Angle of Contact is diminished, it will be yet greater than the Angle of Contact at the Vertex of the cubical Parabola; and therefore any given circular or parabolical Angle of Contact, will be infinitely greater than the Angle of Contact at the Vertex of the cubical Parabola: for that Quantity is infinitely greater than another, which how much soever diminished, it shall always exceed that other.

AGAIN, to the same Axis and Vertex let be described another parabolical Curve AG , whose Ordinates

nates always increase in a subquadruplicate Ratio of the Abscissas; the Angle of Contact FAG will be infinitely less than the Angle FAD : which is easily demonstrated by a way of Argumentation not unlike the former. After the like manner, to the same Axis and Vertex another parabolical Curve AH may be described, whose Ordinates increase in a subquintuplicate Ratio of the Abscissas, in which the Angle of Contact FAH will be infinitely less than the Angle FAG : and so one may proceed *in infinitum*, always assigning more and more parabolical Figures, whose Angles of Contact shall differ infinitely from each other; that is, the Angle FAC will be infinitely less than any rectilinear Angle, and the Angle FAD infinitely less than the Angle FAC , and the Angle FAG infinitely less than the Angle FAD : and so there will be a Series of Angles of Contact proceeding *in infinitum*, whereof every latter Angle is infinitely less than the former; nay, between any two Angles there may be innumerable other Angles inserted, that shall infinitely exceed each other. And also between any two of these Angles there may be inserted an infinite Series of intermediate Angles, whereof each following Angle shall be infinitely less than the foregoing. Moreover, there may be innumerable Angles infinitely greater than the circular Angle of Contact, which yet shall be infinitely less than any rectilinear Angle; and so you may proceed *in infinitum*, nor does Nature know any limits.

I HAVE exhibited these Examples, that our Adversaries may perceive, how prodigiously different their Speculations, concerning the Natures of Things, are from the Things themselves.



LECTURE V.

Of the Subtility of Matter.

HAVING, as we believe, proved by undeniable Arguments, the infinite Divisibility of Matter; and sufficiently answered and refuted the Objections that seemed to carry any weight along with them: it remains, that we consider a little the wonderful Subtility of Nature, and those minute Particles into which Matter is actually divided, or of which it is compounded. It would be very easy, by an abundance of Examples, to place these, as it were, before your Eyes, to expose them to your Senses, and even to shew their Smallness by a Calculation: but we shall produce only a few Instances.

AND first of all, from the great Ductility of Gold, several learned Men, as *Monf. Robault*, in his *Physicks*; our Countryman, the Honourable *Mr. Boyle*, in his Book of *Effluvia*; and lately, the famous *Dr. Halley*, in the *Philosophical Transactions*, N^o 194, have made a Computation of the extreme Smallness of its Parts. *Dr. Halley* has shewn, that one Grain of Gold may be cut into 10000 visible Parts; and therefore since a Grain of Gold is nearly equal to $\frac{21}{100000}$ of a cubick Inch, it follows that a cubick Inch of Gold may be divided into 47 619 047 Parts, which shall be all discernable by the naked Eye.

BESIDES, *Dr. Halley* has computed the Thickness of that very fine Skin of Gold, with which in gilding the Artificers cover Silver-Wire; and has found it not

not exceed the $\frac{1}{124500}$ of an Inch: that is, if an Inch in Length was divided into 124500 Parts, the Thickness of the gilding would scarce exceed one of those Parts; and therefore the Cube of the hundredth Part of an Inch, or which is the same thing, the $\frac{1}{1000000}$ Part of a cubick Inch would contain 243 000 000 such Parts.

THAT noble Philosopher Mr. Robert Boyle, has given, in his Book of *the Nature and Subtility of Effluvia*, many Experiments concerning this Subject; whence we shall here borrow two or three. And, first of all he dissolved one Grain of Copper in Spirit of Salt *Armoniack*, and that Solution being mixed with distilled Water, gave a very deep and conspicuous blue Tincture to 28534 Grains of Water: whence, since the Quantity of Water, whose Weight is one Grain, is equal to $\frac{37}{10000}$ of a cubick Inch, 28534 Grains of Water will be equal in Magnitude to 105, 57 cubick Inches. Since therefore one Grain of Copper can give a blue Colour to so great a quantity of Water, it is necessary that there must be some Part of this Copper in every visible part of that quantity of Water; and therefore as many as are the Parts of that Water visible to the Eye, into so many Parts at least was that one Grain of Copper divided. But a Line is sensible to the Sight whose Length is the hundredth part of an Inch, and consequently the Square or the Cube of that Line, will be much more discernable by the Sight: wherefore, since the Cube whose Side is $\frac{1}{100}$ of an Inch, is the $\frac{1}{1000000}$ part of an Inch, it follows, that at the least in 105, 57 cubick Inches of Water, there are 105 570 000 Parts distinguishable by the Sight: and therefore by this Solution

tion one Grain of Copper was at the least divided into as many Parts. But the Magnitude of a Grain of Copper is equal to about $\frac{55}{100000}$ Parts of an Inch; and therefore, since a cubick Inch contains almost 20000 such Parts, it hence follows, that a cubick Inch of Copper may be actually resolved into 2 111 400 000 000 Parts: and if there be taken the least Grain of Sand, such a one, for example, as its Diameter may be the hundredth part of an Inch, or, which is the same thing, the Grain of Sand itself be the million part of an Inch, this will contain 2 114 000 such Parts, into which the Copper was divided.

THE second Example that we propose, shall be drawn from the following Principles.

ALL the modern Philosophers agree that Odours arise from Effluvia, that proceeding from the odoriferous Bodies, are dispersed on every side into the Medium, and which, by means of the Air we draw up our Nostrils, do rush upon the olfactory Nerves, and so affect the Sensorium; whence it follows, that in whatever place the Odour of any Body is sensible, there must be in that very place some Particles of the odoriferous Body affecting the Sense. But there are many odorous Bodies which are easily smelt at the distance of five feet, and which do there affect the olfactory Sense; there must therefore be some Particles of the odorous Body diffused through all that Space, insomuch that wherever our Nostrils shall be placed in that Space, there, it is necessary, that there be some Effluvia of the odoriferous Body; at least there must be some in that quantity of Air, that we draw through our Nostrils in Inspiration. Let us suppose then, that there is but one Particle of the odorous Body in every Part of that Space, which Part shall be equal to the fourth Part of a cubick Inch; and tho' it is probable,
Effluvia

Effluvia so rare will scarce affect the Sense, yet we shall at present assume no more; so many therefore at the least will be the Particles producing the Odour, as there are Spaces, each of which are equal to the fourth part of an Inch, in a Sphere, whose Semi-diameter shall be five feet; but in that Sphere there will be 57839616 such Spaces: so many therefore will be the Particles producing the Odour in that Space.

HAVING in some measure determined the Number of Effluvia, we shall proceed to discover their Magnitude. Since it is necessary, that a Body must be diminished in Weight, in proportion to the Effluvia that flow from it; the Weight of all the Effluvia proceeding from an odoriferous Body in a given time, will equal the Weight of the Part of the Body that is lost in that time. Now Mr. Boyle has proved by an Experiment, that a certain Mass of *Assa fetida* exposed to the open Air, has lost in six days time the eighth part of a Grain in Weight: but since the Flux of Effluvia from an odoriferous Body is continual, it is manifest that it ought to be proportionable to the Time; and therefore in one Minute's Time the Weight of the Effluvia flowing from the *Assa Fetida*, will be equal to $\frac{1}{69120}$ of a Grain. But the Magnitude of a Particle of Water, whose Weight is one Grain, is equal to $\frac{369}{100000}$ Parts of an Inch; and therefore a Particle of the same Water, whose Weight is $\frac{1}{69120}$ of a Grain, will be equal in Magnitude to $\frac{533}{10000000000}$ of a cubick Inch: but the Gravity of *Assa fetida* to that of Water is (as I myself have experienced) as 8 to 7, and therefore the Magnitude of a Quantity of *Assa fetida*, whose Weight

E

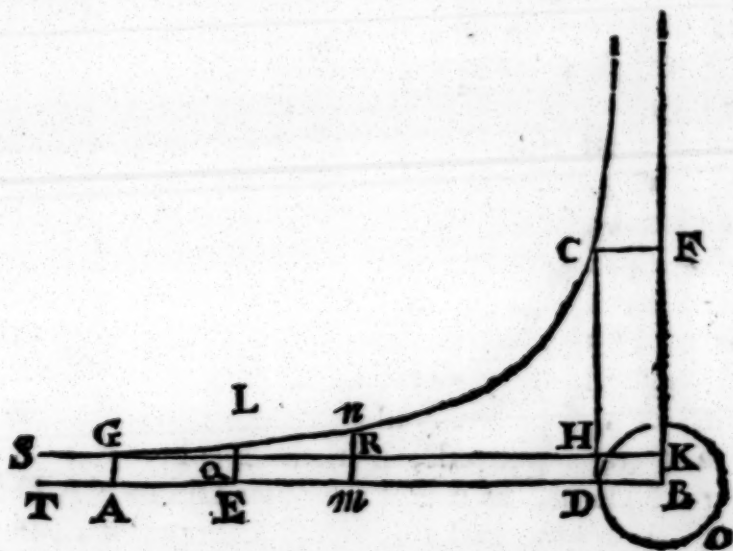
is

is $\frac{1}{69120}$ of a Grain, will be equal to $\frac{466}{10000000000}$ of a cubick Inch. But the Number of all the Effluvia was found before to be 57839616, and therefore since all these Effluvia are only equal to $\frac{466}{10000000000}$ Parts of a cubick Inch, each Particle will be equal to $\frac{466}{578396160000000000}$ Parts of a cubick Inch; or by reducing this Fraction to a Decimal, the Magnitude of each Particle will be equal to $\frac{8}{10\ 000\ 000\ 000\ 000\ 000}$ Parts of a cubick Inch.

WE have hitherto supposed the Particles producing the Odour to be every where equally diffused throughout the above mentioned Distance; but since towards the Center or odoriferous Body, whence they proceed, they are more and denser than towards the outward Superficies of the Sphere, there will be many more Particles than we have before demonstrated. For since Odours (as all other Qualities, that are propagated from a Center in right Lines) decrease in a duplicate Ratio of the Distance increased from that Center, the Number of Particles producing the Odour, and included in a given Space, as, for example, the fourth Part of a cubick Inch, will be, at the distance of one foot, quadruple the Number of Particles placed in the same Space, at the distance of two feet from the Center; and nine times greater than the Number of Particles at the distance of three feet, and so of the rest: but if they were in no place in greater number than at the extreme Superficies, the Number would then be, as before discovered, 57839616. It is therefore manifest, that their Number is far greater than the aforesaid Number.

THAT therefore, in this case, we may determine the Number of the Particles producing the Odour,
we

we ought to know the Quantity of *Assa fetida* that Mr. Boyle exposed to the Air; but we cannot learn this from his Writings. It is therefore necessary we assume some Quantity; but the less we make it, the greater will be, *ceteris paribus*, the Proportion of the Number of Particles proceeding from it, to the Number before found out. That therefore the Number we shall discover may not exceed the Truth, we ought to assume such a Quantity, as probably will be greater than that which Mr. Boyle exposed to the Air: and let it be equal to a Sphere, whose Diameter is six Inches, here represented by the Circle DHO; and let the right Line AD be five feet or sixty Inches; AB will be 63 Inches. To the Point A on AB let be erected the Perpendicular AG,



which let represent the Density or Number of Particles in a given Space at the distance AB; and if at all distances the Density of the Particles was the same, their Number might be expounded by the innumerable right Lines EQ, mR, DH, &c. completing the Parallelogram AH; that is, by the Parallelogram

E a rallelogram

rallelogram AH itself. But since the Number of Particles, in approaching to the Center, is supposed to increase in a duplicate Ratio of the diminished Distance; at the Points E , m , D , and innumerable others taken in the right Line AB , let be erected the Perpendiculars EL , mn , DC , which let be to AG as the Square of the right Line AB to the Squares of the right Lines EB , mb , DB , &c. respectively; and through the Points G , L , n , C , and innumerable others determined after the same manner, let be drawn a Curve Line. Now if AG represents the Number of Particles at the Distance AB , EL will represent their Number at the Distance EB , on a Supposition that the Density of the Particles are reciprocally in a duplicate Ratio of their Distances from the Center; but EQ would denote their Number, if their Density was every where the same. After the same manner, mn expounds the Density of the Particles at the Distance mb ; but mR would represent their Number, if they were uniformly dense. So likewise DC will denote the Number of the Particles placed at the Distance DB ; but if they were in all places equally dense, their Number would be represented by DH . And therefore the whole Multitude of Particles, that flow from the Sphere DBO , and whose Density decreases as they recede from the Center of a duplicate Ratio of the increased Distance, is to their Multitude, if their Density was every where the same, as it is at the outer Distance AD five Feet, as all the right Lines DC , mn , EL , AG , to the right Lines DH , mR , EQ , AG ; that is, as the mixtilineal Area $ADCG$, to the Area of the Rectangle $GADH$.

THE matter therefore is brought to this, that we inquire out the Proportion which the Area $GADC$ has to the Area of the Rectangle AH . But since the Curve $GLnC$ is of such a nature, that the right
Lines

Lect. 5. *Natural Philosophy.* 53

Lines A G, E L, $m n$, D C, ordinately apply'd to the Asymptote A B, are reciprocally as the Squares of the Distances from the Center; the Curve will be of the hyperbolick kind, and the indeterminable Space C F B T S compounded of Elements that are *reciprocal Secundanes*: and therefore that Space, altho' indeterminable, will be perfectly quadruple, and equal to twice the Rectangle C B, by what is demonstrated by Dr. Wallis in his *Arithmetick of Infinites*. And consequently the indeterminable or indefinitely extended Area C D T S will be equal to the Rectangle C B; and in like manner the indefinitely extended Area G A T S will be equal to the Rectangle G B. The Excess therefore, whereby the Area C D T S exceeds the Area G A T S, will be equal to the Excess, whereby the Parallelogram C B exceeds the Parallelogram G B. Let us therefore investigate the Difference of these Rectangles: Since by Hypothesis A D is 60 Inches, and B D 3, A B will be 63 Inches; and let A G be Unity, and since it is as D B q to A B q so A G to C D, that is, as 9 to 3969; C D will be 441 such Parts, whereof A G is 1: and therefore C D $\times$ D B, or the Rectangle C B will be to the Rectangle B G, as 1323 to 63; and so the Difference of the Rectangles, that is, the Area G H D C will be 1260 such Parts, whereof the Rectangle A H is 60. And therefore the Number of Particles proceeding from the *Assa fatida*, whose Densities decrease in a duplicate Ratio of the increased Distances, and contained within a Sphere whose Diameter is five Feet, is to their Number, (if their Density was every where equal to what it is at the Distance of five Feet) as 1260 to 60; that is, as 21 to 1. If therefore the above-discovered Number 57839616 be multiplied by 21, the Product will give the Number of Particles proceeding from the *Assa fatida*, viz. 1214631936. Besides, if the Fraction

$\frac{8}{10\ 000\ 000\ 000\ 000\ 000}$, which expressed the Magnitude of the Particles in the former case, be divided by 21, the Quotient $\frac{8}{210\ 000\ 000\ 000\ 000\ 000}$ or $\frac{38}{1\ 000\ 000\ 000\ 000\ 000\ 000}$ will exhibite the true Magnitude of each Particle in this second Case.

ALL these things follow from hence, that we suppose a Man able to smell *Affa fatida* at the distance of five feet; but there are other Animals, whose Sense of smelling far exceeds that of a Man, as Hounds, who perceive the Effluvia of wild Beasts remaining on the ground, a great while after the Beast has left the place; and some Birds, that get the Scent of Gun-powder at a great distance. The Subtility of these Effluvia must certainly be much greater than that, which we have discovered by the foregoing Calculation; but for want of Experiments we cannot easily reduce it to numbers.

THE Philosophers, that they may still farther shew the Subtility of Matter, produce the Example of those *Animalcula*, that are observed floating in various Liquors, and the Seed of Animals; these indeed are so small in some Fluids, that they appear like Points, through Microscopes that magnify the Object very much. And that curious Observer of Nature, Mr. *Lewenboeck* has discovered more Animalcules in the Milt of one Cod, even than there are Men at present living on the face of the Earth. But let us discover the Magnitude of these Animalcules: towards which, we shall borrow from Opticks what follow. First, the Image of any Object appears under the same Angle, at the Vertex of Emerision of the Lens, as the Object itself appears under at the Vertex of Incidence; this is demonstrated by Dr. *Gregory*, in the 18th Proposition of his *Elements of Dioptricks*. Secondly, it is found by experience, that those Objects

that

that appear as Points, that is, whose Parts the Sight is not able to distinguish, are seen under an Angle not exceeding a Minute. Thirdly, it is certain by frequent Observation, that most of those Animalcules are of so small a Magnitude, that they appear thro' a Lens, whose focal Distance is the tenth part of an Inch, as so many Points; that is, their Parts cannot be distinguished; and therefore they appear from the Vertex of that Lens, under an Angle not exceeding a Minute. We are now to investigate the Magnitude of an Object that appears under a given Angle at a given Distance: In the present

Case, let C be the Vertex of the Lens, A B the Length of the Animalcule, B C its distance from the Lens equal to $\frac{1}{10}$ of an Inch, and the Angle B C A under



which it is seen at that Distance equal to one Minute: from B C and the Angle B C A being given, we are to find A B the Length of the Object. Now in the rectangular Triangle A B C, there being given (besides the right Angle at B) the Angle B C A one Minute, and the Side B C equal to the tenth part of an Inch, by Trigonometry the Side A B will be found nearly equal to $\frac{3}{100\ 000}$ of an Inch. If there-

fore these Animalcules were of a cubick Figure, that is, of the same Length, Breadth and Thickness, their Magnitude would be expressed by the Cube of the Fraction $\frac{3}{100\ 000}$; that is, by the Number

$\frac{27}{1000\ 000\ 000\ 000\ 000}$; that is, to so many parts of an Inch is each Animalcule equal.

HENCE, what some Philosophers have dreamed concerning Angels, is true of these Animalcules, viz,

that many thousand of them may dance on the Point of a small Needle.

HENCE also we may gather, how vast the Distance is betwixt the smallest floating Animals and the largest sort, how little Proportion the first have to the huge Whales, that appear in the Ocean like Mountains, as oft as they raise their Heads above the Water. For there are in some Liquors Animalcules so small, as upon a Calculation, the whole Magnitude of the Earth is not large enough to be a third proportional to those minute floating Animals and the vast Whales of the Ocean; so that the Earth itself, however big it seems, is found to bear a less proportion to these large Fishes, than they do to those lesser ones, that are beheld through a Microscope, floating in the Seed of Animals.

SINCE every Animalcule is an organized Body, let us a little consider, how delicate and subtile ought the Parts to be, that are necessary to constitute it, and to preserve its vital Actions. Certainly it is not easy to conceive, how it is possible there should be contained in so narrow a compass, the Heart that is the Fountain of its Life, the Muscles necessary to its Motions, the Glands for the Secretion of its Fluids, the Stomach and Bowels to digest its Food, and other innumerable Members, without which it is impossible an Animal should subsist. But since every one of these Members is also an organical Body, they must have likewise Parts necessary to their Actions. For they consist of Fibres, Membranes, Coats, Veins, Arteries, Nerves, and an almost infinite Number of fine Tubes like to these, whose Smallness seems to exceed the very Force of the Imagination. But there are some Parts that ought to be almost infinitely less than these, as the Fluids that flow along these fine Tubes: and such are the Blood, Lymph, and Animal Spirits, whose Subtility even in large Animals is incredible.

LET

LET us then consider the most gross Parts of the Blood in these Animalcules, to wit, the Globules that swim in the Blood, and let us endeavour to reduce their Magnitude to a Calculation.

TOWARDS the effecting of which, we shall make use of the following Hypothesis; *viz.* That the similar solid Parts of different Animals, that is, the similar corporeal Particles, or the Parts consisting of three Dimensions, are as the Magnitudes of the respective Animals. Whence it follows, that the similar linear Dimensions of different Animals, are in a subtriplicate Ratio of the Magnitudes of the Animals; that is, as the cubick Roots of these Animals: as, for example, the Heart of a Man is to the Heart of any Animalcule seen through a Microscope, as the Body of that Man is to the Body of the Animalcule; and therefore, if the Hearts of both are similar Bodies, the Diameter of the one will be to the Diameter of the other, as the cubick Root of the Magnitude of one, to the cubick Root of the other's Magnitude. So likewise the smallest Blood-Vessels in a Man, are to the like smallest Vessels in an Animalcule, as the Magnitude of the Man to the Magnitude of that Animalcule; and the Diameter of a capillary Vessel in the Body of a Man, is to the Diameter of the like Capillary Vessel in the Body of an Animalcule, as the cubick Root of the Magnitude of the Man to the cubick Root of the Magnitude of the Animalcule.

LET us now suppose the mean Magnitude of a Man to be three cubick Feet, or 5184 Inches; as therefore the Magnitude of a middle-sized Man, or 5184 cubick Inches, is to the Magnitude of the Animalcule, found out above, *viz.* $\frac{27}{1\ 000\ 000\ 000\ 000\ 000}$

Parts of a cubick Inch, so are the capillary Vessels in a human Body to the like capillary Vessels in an Ani-

Animalcule; and as the cubick Root of the Magnitude of a Man, or the cubick Root of the Number 5184 to the cubick Root of the Magnitude of an Animal, or to the cubick Root of the Number

$\frac{27}{1\ 000\ 000\ 000\ 000\ 000}$, that is, nearly as 17 to $\frac{3}{100\ 000}$, so

is the Diameter of a capillary Vessel in a human Body to the Diameter of a capillary Vessel in an Animalcule. But Mr. *Lewenboeck* has by the help of a Microscope discovered Vessels in a human Body so small, that if the Diameter of a Grain of Sand be supposed equal to $\frac{1}{30}$ of an Inch, it will contain 2640 Diameters of the Vessels which he found out in the Body of a Man; and therefore the Diameter of one

of these small Vessels will be equal to $\frac{1}{2640} \times \frac{1}{30}$ of an

Inch, that is, equal to $\frac{1}{79200}$ Parts of an Inch.

And although it is certain these Vessels were not the least of all that are in a human Body, for that there must be others much less than these, is easy to be shewn; but however we shall suppose these to be the

least of all. Let it be therefore as 17 to $\frac{3}{100\ 000}$, so

$\frac{1}{79200}$ to another Number; which Number will express in the parts of an Inch, the Diameter of the

smallest Vessel in an Animalcule: which working

by the Rule of Three, is found to be $\frac{3}{134\ 640\ 000\ 000}$,

this Fraction reduced to a Decimal, will be nearly

$\frac{22}{1\ 000\ 000\ 000\ 000}$ or in round Numbers) $\frac{2}{100\ 000\ 000\ 000}$,

But since it is necessary that the Diameter of the Globule, or the fluid Particle, that is contained in any Vessel, be not greater than the Diameter of that Vessel; the Diameter of a Globule of Blood that flows

through these smallest Vessels, will not be greater than $\frac{2}{100\ 000\ 000\ 000}$ Parts of an Inch. And therefore the Solidity or Magnitude of these Globules will be less than the Cube of that Diameter, that is, less than $\frac{1}{1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}$ Parts of a cubick Inch: that is, the Magnitude of a Globule is less than that part of a cubick Inch, which is expressed by a Fraction, whose Numerator is eight, but its Denominator a Number consisting of a Unit with thirty and three Cyphers after it.

SINCE the Fraction, that expresses the Magnitude of these Globules, consists of so many Cyphers, that their true Quantity cannot thence be readily conceived; we shall proceed farther, and compare these Globules with such other small Bodies as are visible to the naked Eye, viz. with the smallest Grains of Sand, such, for example, as their Diameters do not exceed the hundredth part of an inch: and lastly, we shall compare these small Grains of Sand with other great Bodies of the Earth, as, for example, huge Mountains; that we may perceive what proportion they bear to one another: and so we shall the more easily comprehend the Smallness of Particles. But why do I make use of this word? since I should rather say, by this Comparison their Subtility will appear incomprehensible. For it may be gathered from thence, that Ten thousand two hundred and fifty six of the highest Mountains in the whole Earth do not contain as many Grains of Sand, as one Grain of Sand can of the Blood-Globules of these Animalcules. It is no wonder, if you here stand amazed, and being struck with so prodigious a thing, should call in question the infinite Divisibility of Matter, although it is supported by uncontrollable Demonstrations. But however incredible this
 Affec-

Assertion may appear at first sight, we shall notwithstanding deduce it from easy and evident Principles.

THAT our Calculation may proceed the easier, we shall call the tenth part of a Foot an Inch, and suppose that if an hundred Grains of Sand were placed one by another, they would occupy the Space of an Inch in length; or, which is the same thing, let a thousand contiguous Grains of Sand be supposed to be extended through the Length of one Foot, there will therefore be in one cubick Inch 1 000 000 Grains of Sand, and in a cubick Foot there will be 1 000 000 000 Grains. Let a Mile or 1000 Paces be equal to 5000 Feet, there will be then 125 000 000 000 cubick Feet in a cubick Mile: so that the Number of Grains of Sand, that can be contained in a cubick Mile, will be 125 000 000 000 000 000 000.

Now that we may have the Dimensions of the Mountains, we will make choice of the highest, as it is supposed, of the whole Earth, viz. that which is in the Island of *Teneriff*, and called *El Pico de Terrario*, whose perpendicular Altitude is vulgarly esteemed three *Italian Miles*. We will suppose the Figure of this Mountain to be a Cone, and its Circumference at the Base to be five and thirty Miles, the Area of the Base will be about 97,5 Miles; for as 314 to 100, that is, as the Circumference of a Circle to its Diameter, so is 35 to 11,14 the Diameter or Thickness of the Mountain at its Base; 27,85 the fourth of which being multiplied by the Circumference 35, gives the Area of the Base, viz. 97,5 square Miles. Since therefore the Mountain is, by Hypothesis, of a conick Figure, if its Base be multiplied into a third part of its Height, the Product will give the solid Content of the Mountain; and the third part of the Height is, by Supposition, equal to one Mile, which multiplying the Number 97,5, the Product or Solidity of the Mountain

Lect. 5. *Natural Philosophy.*

61

Mountain will be equal to 97,5 cubick Miles: which Number, if it be again multiplied by 125 000 000 000 000 000 000, the Product or Number 12187 500 000 000 000 000 000, will exhibite the Number of Grains of Sand, of which the Mountain of the Island *Teneriff* might be composed.

THESE being found out, let us see how many Blood-Globules may be contained in one Grain of Sand. From what has been shewn before, the Magnitude of each Globule is less than the

8

1 000 000 000 000 000 000 000 000 000 000 Parts of an Inch; and the Magnitude of a Grain of Sand is equal to the $\frac{1}{1\ 000\ 000}$ Part of an Inch: so that if the latter Number is divided by the former, the Quotient $\frac{1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}{8\ 000\ 000}$, or $\frac{1\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000\ 000}{8}$; that is,

125 000 000 000 000 000 000 000 000 000 is less than the Number of Blood-Globules that may be contained within the Magnitude of a Grain of Sand: but this Number 125 000 000 000 000 000 000 000 000 000 divided by 12 187 500 000 000 000 000 000 000, the Number of Sands that may be contained in the Mountain of the Island *Teneriff*, the Quotient will be greater than the Number 10256. So that one Grain of Sand may contain ten thousand two hundred and fifty six times more Blood-Globules in it, than the highest Mountain of the whole Earth does Grains of Sand; or, which is the same thing, ten thousand two hundred and fifty six Mountains, each of which shall be equal to the highest Mountain on the whole Earth, cannot contain in them as many Grains of Sand, as one Grain of Sand may contain in itself of sanguineous Particles of Animalcules, that are by a Microscope seen

seen floating in some Fluids: which was to be shewn. Since therefore these Globules are of so small a Magnitude, what must we think of the Particles composing the Fluid in which the Globules are carried, and of the Subtility of the Animal Spirits? This, no doubt, is so great, as to exceed all Calculation, and even the Force of the Imagination.

THIS Subtility of Nature is wonderful beyond measure; but there are other Particles of Matter still more subtile than these, to which if the above-mentioned Globules were compared, they would not only appear as Mountains, but as vast Earths. I mean the Particles of Light, which are darted all around from the lucid Body, with an inexpressible Celerity, whose Subtility the human Mind will perhaps be never, unless when it shall be made perfect in the Heavens, able thoroughly to comprehend. That it is immense, may hence be gathered, that the Light of the smallest Candle, in a time altogether insensible, and without any discernible Diminution of the Candle, may be perceived by the Eye, at the distance of two Miles; whence it is necessary, that in every assignable Part of the Sphere of Activity of that Candle, the Diameter of which Sphere is greater than four Miles, and in every assignable Portion of Time, there are some Particles of that Light, which enter, or are ready to enter the Eye; which will be different in different Parts of Time. And from this inef-
fable Subtility of Light it is, that the Sun, although from the Beginning of its Creation, it has continually emitted Light, and that with great Swiftnefs; yet in all that time has not lost any thing sensible of its Magnitude, notwithstanding it daily, tho' inconsiderably, decreases in Quantity: whence, tho' after six thousand Years, its Diminution is not yet become remarkable, notwithstanding after a finite Series of Years, altho' a very protracted one, it will be wholly
diffi-

diffipated. Whence it follows, that this World can neither exist to Eternity, nor could it have existed from Eternity.

FROM the Demonstration of the infinite Divisibility of Matter are derived the following Theorems, relating to its Rarity, and the Tenuity of its Composition.

LEMMA.

ANY Quantity of Matter being given, of it, or any part of it, a concave Sphere may be formed, whose Semidiameter shall be equal to any given right Line.

LET the Particle of Matter be a^3 , and the given right Line be b . The Ratio of the Circumference of a Circle to its Radius let be as p to r . Let the Semidiameter of the Concavity be called x , the Thickness of the Shell encompassing the Spherical Concavity, will be $b - x$, and the Cylinder circumscribed about the Spheres whose Radius is b , will be $\frac{p \times b^3}{r}$, whence the Sphere inscribed within the Cylin-

der will be $\frac{2 \times p b^3}{3 r}$; by the same reason, the Sphere

whose Radius is x , will be $\frac{2 \times p x^3}{3 r}$, whose Difference

$\frac{2 p}{3 \times r} \times b^3 - x^3$ is to be made equal to the spherical Shell,

or given Particle of Matter: that is, it will be

$\frac{2 p}{3 r} b^3 - x^3 = a^3$ or $b^3 - x^3 = \frac{3 r a^3}{2 p}$ whence $x^3 = b^3 -$

$\frac{3 r a^3}{2 p}$ and $x = \sqrt[3]{b^3 - \frac{3 r a^3}{2 p}}$. So that the Thickness

of the Spherical Shell, or $b - x$, will be $= b -$

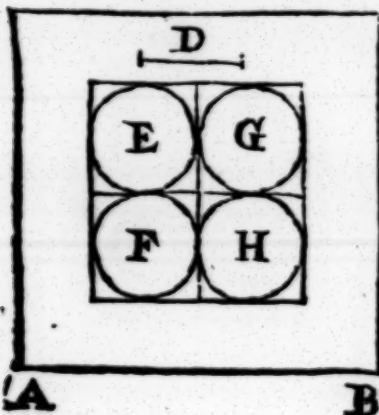
$\sqrt[3]{b^3 - \frac{3 r a^3}{2 p}}$.

By

By the same reason may be made of a given Quantity of Matter, concave Cubes, concave Cylinders, or concave Bodies of any other figure whatsoever, whose Sides shall be equal to a given right Line.

The First THEOREM.

GIVEN any Quantity of Matter, however small, and given any finite Space, however large; which, for example, let be a Cube, that shall circumscribe the Orb of Saturn: it is possible that the Matter of that Grain of Sand may be diffused through all that Space, and so fill it, that there shall be in it no Pore, whose Diameter will exceed a given right Line.



LET there be a given Cube Space, whose Side let be the right Line A B, equal to the Diameter of Saturn's Orbit; and let be given a Particle of Matter, whose Quantity let be b^3 , and let the given right Line (greater than which the Diameter of the Pores ought not to be) be D. Let the right Line A B be conceived to be divided into Parts equal to the right Line D, whose Number will be finite, when neither the right Line A B is supposed infinitely great, nor the right Line D infinitely small. Let the Number be n , that is, let be $n D = A B$, so that it will be $n^3 D^3$ equal to the Cube of the right Line A B. Let also the given Space be conceived to be divided into Cubes, each of whose Sides are equal to the right Line D, and the Number of the Cubes will be n^3 ; and these Cubes will be represented in the Figure by the Spaces E, F, G, H. Again, let the

the Particle b^3 be supposed to be divided into Parts, whose Number let be n^3 , and in each cubick Space let be placed one of those Particles; and by this means the Matter b^3 will be diffused through all that Space. Besides, each Particle of the Matter b^3 being placed, as it were, in its Cell, may be formed into a concave Sphere, whose Diameter may be equal to the given right Line D; whence it will follow, that each Sphere will touch that which is next to it; and the given Particle of Matter, however small b^3 , will so fill the given Space, that there will be no Pore in it, whose Diameter will exceed the given right Line D. Q. E. D.

Cor. THERE may be a given Body, whose Matter if it be reduced into a Space absolutely full, that Space may be any given Part of the former Magnitude.

The Second THEOREM.

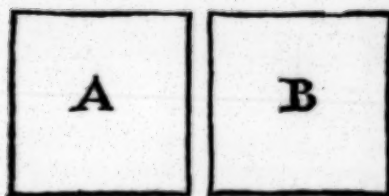
THERE may be two Bodies equal in Bulk, whose Quantities of Matter may be very unequal, and tho' they have any given Ratio to one another, yet the Sums of the Pores or empty Spaces in the Bodies, may almost approach to a Ratio of Equality. Or, in the Cartesian Stile; All the Space, that is possessed by a subtile Matter within the Pores of one Body, may be almost equal to the Space that is occupied by the like Matter within another Body; although the proper Matter of one Body exceeds ten thousand or an hundred thousand times the proper Matter of the other Body, and the Bodies may be equal in Bulk.

LET there be, for example, a cubick Inch of Gold, and a cubick Inch of Air not condensed. It is certain the Quantity of Matter in the Gold will exceed twenty thousand times the Matter of Air; yet it may be, that the Spaces in the Gold, either absolutely empty, or replete with a subtile Matter,

F

shall

shall be almost equal to the Spaces in the Air, either empty, or replete only with a subtile Matter.



LET A and B be two Bodies equal in Magnitude ; and let both, for example, be a Cube of one Inch. And let the Body A be ten thousand times heavier than the

Body B, whence the Body A will exceed the Body B in Quantity of Matter ten thousand times. Let us now suppose the Quantity of Matter in A to be reduced into a Space absolutely full, which let be the hundred thousandth part of a cubick Inch ; (which is possible from the Corollary of the precedent Theorem.) Whence since the Matter in A exceeds the Matter in B ten thousand times, that Matter in B, if it be reduced into a Space absolutely full, will possess only the $\frac{1}{1\,000\,000\,000}$ Part of a cubick Inch ; so that the rest of the Parts 999 999 999 will be either absolutely empty, or replete only with some subtile Matter, such as the *Cartesians* suppose. Again, since the Quantity of Matter in A only fills the hundred thousandth Part of an Inch, there will be in the Body A 99999 hundred thousand Parts, either empty or replete with a subtile Matter ; that is, by reducing the Fraction to the Denominator of the former Fraction, there will be in A 999 990 000 empty Parts. And therefore the Vacuities in A will be to the Vacuities in B, as the Number 999 990 000 to the Number 999 999 999 ; which Numbers are almost to one another in the Ratio of Equality, for their Difference bears but a small Proportion to the Numbers themselves. And therefore the empty Spaces, or those only replete with a subtile Matter, which are in the two Bodies A and B, obtaining to
one

one another the same Ratio as those Numbers, are also almost in a Ratio of Equality. *Q. E. D.*

BUT that all Bodies are very rare, that is, contain but a very small Quantity of Matter in respect to their Bulk, is most certain from the Properties of diaphanous Bodies; for the *Rays of Light* within Glass or Water are diffused in right Lines, as well as in Air; whatever Side of the diaphanous Body is exposed to the Light: And therefore from any the least assignable Part of the diaphanous Body, to any other Part of it, there is always extended in these Bodies a rectilinear Pore, through which the Light may pass; and this cannot be, unless the Matter of the diaphanous Body obtains but a very little Proportion to its Bulk; and perhaps the Quantity of Matter in Glass has not a greater proportion to its Magnitude, than a Grain of Sand to the whole Bulk of the Globe of the Earth: but that this is not impossible, we have shewn above. Whence, since Gold is not eight times denser than Glass; its Matter also, to its proper Bulk, bears but a very small proportion.

HENCE may be gathered the Reason why the magnetick *Effluvia* pervade with the same facility both dense Gold and the more rare Air.

FROM these Propositions likewise, and from the great Swiftneſs of Light, may be gathered the Reason, why the *Rays of Light* proceeding from many Objects, and transmitted through a small hole, do not mutually hinder one another, but their Motion is continued in a right Line; which can scarce be explained by the Motion or Impulse of a Fluid, constituting a *Plenum*: for any Body impelled by many Powers at the same time, in different Directions, will receive only one, and a determinate Direction, compounded of them all.



LECTURE VI.

Of Motion, Place, and Time.

SINCE we have hitherto sufficiently treated of the Solidity, Extension, Divisibility and Subtility of Bodies; we now come to explain Motion, the noblest Affection that Body is endued with: by the mediation whereof, Nature discovers herself acting in that Variety of Things, which ought not to be beheld without Wonder and Astonishment; and without which, all the Ornament and Beauty of the World would perish, and a horrid Darknefs and an infinite Numbnefs would possess every thing. On this depend the Vicissitudes of Days and Nights, and the so great Variety of Cold and Heat, Snow, Rain, and Sun-shine succeeding each other, and all the Seasons of the Year. By Motion Plants grow, Trees are nourished, and Animals live; since Life itself consists only in Motion, that is, the Circulation of the Blood. But why do I spend time in enumerating Particulars? since all things owe their Birth to Motion.

THE Science therefore of Motion is so necessary to philosophize aright, that not the least Operation of Nature can be investigated without it. Hence the famous and most true Saying of the Philosopher, *Αναγκαῖον ἀγνοομένης αὐτῆς κινήσεως ἀγνοεῖσθαι καὶ τὴν φύσιν*, Motion being unknown, Nature must of necessity be so likewise.

THE Philosophers, or rather the Metaphysicians, have had various Disputes concerning the Nature, Causes, and Communication of Motion; and the Con-

Confusion of Ideas, and the Darkneſs that thence aroſe, appear to have been ſo great; that amongſt the Follies of Diſputation, the natural and ſimple Notion that they had of it, ſeems to have been loſt amongſt them. For there can ſcarce be found one of the common People, or the meaneſt Artificer, but he knows more of the true Nature and Cauſe of Motion than all theſe wrangling Philoſophers together, ſome of which were arrived to that height of Folly, as to deny all Motion, as a thing impoſſible, to Bodies; and they propoſed ſome Cavils, whereby they flattered themſelves that they had demonſtrated its Impoſſibility.

We will produce here ſome of their ſtrongeſt Arguments: and the firſt ſhall be that of *Diodorus Cronus*; namely, If a Body moves, it muſt either move in the place where it is, or in the place where it is not, both which are impoſſible: for if it moves in the place where it is, it will never depart from that place, and ſo there will be no Motion; in like manner it cannot move in the place where it is not, becauſe nothing acts in a place where it is not; therefore a Body will not move at all. I anſwer, that a Body neither moves in the place where it is, nor in the place where it is not, but it moves from one place to another.

THE ſecond Argument ſhall be that of *Zeno*, to which he gave the Name of *Achilles*: and thereby he endeavours to prove, that if there be ſuch a thing as Motion, *Achilles*, though ever ſo ſwift, could never overtake a Tortoiſe, though the ſloweſt of Animals. The Argument is this; Suppoſe *Achilles* to be diſtant from the Tortoiſe a certain finite Space, as, for example, a Mile, and let us ſuppoſe he moves an hundred times faſter than the Tortoiſe; therefore whiſt *Achilles* runs one Mile, the Tortoiſe has moved forwards the hundredth part of a Mile; ſo that *Achilles* has

not yet overtaken the Tortoise: And again whilst *Achilles* passes over that hundredth part of a Mile, the Tortoise in the mean while will have crept through the ten thousandth part of a Mile; insomuch that neither has *Achilles* yet overtaken the Tortoise. After the same manner, whilst *Achilles* runs that ten thousandth part of a Mile, the Tortoise will have advanced forward the million part of a Mile; so that *Achilles* has not yet come up with the Tortoise. And so he may go on *in infinitum*, nor will he ever be able to overtake the Tortoise, but there will be always some distance betwixt *Achilles* and the Tortoise.

THIS is the famous Argument of *Zeno*; to answer which, some have wrote whole Treatises: but we shall easily dissolve the Knot, by saying, That a Mile, together with the hundredth part of a Mile, together with the ten thousandth part of a Mile, with the million part of a Mile, and so on *in infinitum*, will be equal to a finite Quantity. For it is demonstrated by the Arithmeticians, that the Sum of any Series of Quantities decreasing in any Geometrical proportion *in infinitum*, will be equal to a finite Quantity; but the $\frac{1}{100}$ part of a Mile, together with the $\frac{1}{10\,000}$ part, together with the $\frac{1}{1\,000\,000}$ part, together with the $\frac{1}{100\,000\,000}$, or the hundred million part, and so on *in infinitum*, is a Series of Quantities decreasing in Geometrical Proportion *in infinitum*: so that its Sum, since it is equal to a finite Quantity, may be run over by a Body moving with a given Velocity, in a finite time. For let us suppose *Achilles* in the space of an Hour to run a Mile, and therefore he will pass over the hundredth part of a Mile in the hundredth part of an Hour, and the ten thousandth part of a Mile in the ten thou-

thousandth part of an Hour; and after the same manner he will move through the million part of a Mile in the million part of an Hour, and so of the rest. If therefore one Hour, together with the hundredth part of an Hour, together with the ten thousandth part of an Hour, together with the million part of an Hour, $+\frac{1}{100\ 000\ 000}$, &c. *in infinitum*; if I say, the Sum of this Series continued *in infinitum* is equal to an infinite Space of Time, it is certain *Achilles* would never overtake the Tortoise in any finite time: but since, as we said before, the $\frac{1}{100} + \frac{1}{10\ 000} + \frac{1}{1\ 000\ 000}$ &c. Parts of an Hour, is a Series of Quantities decreasing in Geometrical Proportion *in infinitum*; its Sum will be equal to a finite Quantity, *viz.* the ninety-ninth part of an Hour, as might be easily demonstrated. And within that Space of Time, all the Particles of Time, however infinite in Number, will be passed by. We say therefore, that *Achilles* will overtake the Tortoise, after one Hour, and all those Particles of Time contained in the foregoing Series, and which are infinite in Number, are expired; that is, he will arrive at the Tortoise, after one Hour and the ninety-ninth Part of an Hour: And so the Force of this Argument is destroyed, tho' its Patrons have so often boasted of it as unanswerable.

THIS Argument is also wont to be brought against Motion: Let B and C be two contiguous Points, and the Body A be moved from B to C, in an Instant D: when A is moved, it is supposed to be in B, and therefore in that Instant it cannot arrive at C, namely, because it is supposed to be in B; and it cannot be in both in the same Instant, because nothing can be at once in two places, that is, in the same Instant; and therefore in the Instant in which

it is in B, it cannot arrive at C, because it is yet supposed to be in B; and consequently, according to the Authors of this Argument, it will never come to C.

It is easy to answer to this Argument, by saying that at the Beginning of the Instant D, indeed H is in the Point B, but at the end of it, in C; for that Time wherein any finite Motion is performed, must have a beginning and an end.

BUT besides, in this Argument there are not a few things assumed, which are false and impossible: as, for example, when two Points are supposed contiguous. If by a Point is meant an indivisible Part, or the least Quantity, we have before demonstrated, that there are not given such Points; and therefore if the Argument rests on this Hypothesis, it will be impossible it should have any force on the human Mind, against the Existence of Motion. But if by Points are meant mathematical Points, such, for example, as are the Terminations, Intersections and Contacts of Lines, these indeed we acknowledge as possible, however it is impossible that any thing should move in them: for whatever moves, it must move through a Space; but a mathematical Point contiguous to another Point, cannot make a Space, but a Point only. For as in Arithmetick a thousand Cyphers, or Nothing taken a thousand times, still is equivalent to Nothing; so in Geometry a thousand Points, or indeed an infinite Number of them together, do not compose a Quantity, but are equivalent to a Point, or no Quantity. Whence, since two contiguous Points are but equal to a Point, I readily grant that Motion cannot be through them: but nothing absurd follows from thence, for Motion through a Space is not taken away, but Motion through a Point; and it would be absurd indeed to grant such a Motion to be.

WHAT

WHAT we have said of Points, the same may be accommodated to Instants or Moments of Time, by shewing that as all Magnitudes, so likewise Time is divisible *in infinitum*; so that there is no Particle of Time that can be properly called an Instant or Point of Time: as no Part of a Line coincides with a Geometrical Point, and as infinite Points do not compose a Line, but a Point, so likewise infinite Moments, or Points of Time, are equal to no Time. An Interval indeed of Time betwixt different Moments, may be equal to a given Time, but the Moments themselves will be equal to no Time; for Time is not compounded of Moments, but of Parts, which are also Times, nor is Motion performed in an Instant, but in Time.

BUT leaving these Trifles, we return to our Design.

SINCE the Motion, of which we are to treat, is local Motion, our Purpose requires, that we first deliver some things concerning Place and Time. Place is wont to be distinguished into internal and external. The internal Place is the Space that is filled by the Body which is said to be in that Place; but the external Place is that alone, which by *Aristotle* is defined, and called the concave Superficies of the ambient Body, and which contains the Body that is said to be there placed.

BUT perhaps Place, as well as Space, is more clearly distinguished into absolute and relative. Absolute or primary Place is that part of the immovable, permanent, and every way expanded Space, which is taken up by the Body there placed. Relative or secondary Place is that apparent and sensible one, which is discerned by our Senses from its Situation in respect of other Bodies. For since Space itself is a similar and uniform Being, whose Parts cannot be seen, or distinguished from one another; therefore it is fitting that the Places of Bodies be referred

referred to other Bodies, and determined by their Distances from, and Positions to, other Bodies : as, for example, let us suppose any one to sit in the Corner of some House ; his place will be defined by the Distance, Respect, and Position, that he has to other Corners, Walls, and surrounding Bodies, that are looked upon as immoveable : and as long as he keeps the same Distance and Situation from these Bodies, so long is he supposed to remain in the same Place. So likewise if any one sits in a Ship, whether the Ship moves or not, as long as he keeps at the same Distance from all the Parts of the Ship, which are looked upon as being at rest, and his Position to them all remains the same, his relative Place will also remain the same.

WHAT we have said of Place, may be in like manner applied to Space ; for that may be distinguished into absolute and relative. We call that absolute, which of its own nature, and without relation to any thing besides, always remains similar and immoveable. But that is relative which is referred to some Bodies, by which it is determined and measured ; whose Parts, to wit, always keep the same Position and Situation to those Bodies, and whose Distance from them (*viz.* Bodies) always remains unchanged and the same.

RELATIVE Space is always the same with absolute Space in Magnitude and Figure, but however it is not necessary that it should be always numerically the same Space : for if we suppose the Ship to be moved, the absolute Space contained within the Cavity of the Ship, will be different in different Places ; but since the Cavity and Figure of the Ship remain the same, the Magnitude of the Space contained in it, and its Figure will be the same, and its Parts alike situated will always have the same Position and Distance in respect to the same Parts of the Ship : and there-

therefore it ought to be called the same relative Space.

So likewise in the Hypothesis of the Earth's Motion, the Space that is contained within the Walls of any Building, altho' by considering it as absolute, always is changed; yet since the Cavity of the Building remains the same, there is the same Figure, and all the similar Parts of the contained Space always keep the same Situation to the same Parts of the Building; and since they always observe the same Position to the relative Space of our Air, as also to all the Parts of the Earth, that Space may be called relative.

AND after the same manner may Time be distinguished into absolute and relative. Absolute Time flows equally, that is, it never proceeds faster or slower, but without any relation to the Motion of Bodies, it glides along with an equal Tenour. Relative or apparent Time is the sensible Measure of any Duration by the means of Motion; for since the equal Flux of Time itself does not affect the Senses, there must be called in the help of an equable Motion, as some sensible Measure, which may determine its Quantity, whose Parts may always correspond and be proportionable to the Parts of Time. But that uniform Motion, which is to be applied to the Measure of Time, ought to be the most remarkable, obvious to every one, and affecting the Senses of all, such as are commonly thought to be the Revolutions of the Sun, and Moon, and Stars: by which we divide Time into Hours, Days, Months, and Years. And as we judge those Times to be equal, that pass whilst a moving Body, carried by an equable Velocity, runs over equal Spaces; so likewise those Times are said to be equal, which flow whilst the Sun or Moon perform their Revolutions, that to our Senses are equal.

BUT

BUT since, as we said before, the Flux of Time cannot be accelerated or retarded, but all Bodies are moved sometimes faster and sometimes slower, nor perhaps is there given in nature a perfectly equable Motion; it is necessary that absolute Time should be something truly and really distinct from Motion, nor does its Nature more depend on the Motion of Bodies than on their Rest. For let us suppose the Heavens and the Stars to have remained without Motion from the very Creation of the World, it does not thence follow that the Course of Time would have been at a stand, but the Duration of that quiescent State would be equal to the very Time that is now elapsed. Besides, since we learn from the Scriptures, that in the days of *Joshua* the Sun remained for some time immoveable in the same part of the Heavens, yet absolute Time was not therefore at a stand, and began again its Course with the Sun, but flowed in the same Tenour as before, altho' all the Sun-Dials shewed the same Hour of the Day, during all the time of that Station: and so indeed the apparent Time, *viz.* that related to the Motion of the Sun, stood still, when in the mean while the absolute Time proceeded uniformly on.

So likewise since at present the apparent Motion of the Sun is not uniform, neither will its diurnal Revolution be equable, as all the Astronomers acknowledge, but sometimes it proceeds in a swifter, sometimes in a slower degree; and therefore the natural Day, *ὑπερήμερον*, or the Space of Time elapsed in one diurnal Revolution, will be sometimes less, sometimes greater: so that the apparent Time does not proceed in the same Tenour as the absolute, whence it ought to be distinguished from it.

SINCE absolute Time is a Quantity uniformly extended, and in its own nature most simple, it may be aptly represented and proposed to our Imagination

tion by the most simple Magnitudes : such, in the first place, seem to be right Lines and Circles, with which and Time there is a certain Analogy. For all the Parts as well of Time as of right Lines and Circles, are every where similar and uniform ; and as a Line is generated by the Motion or Flux of a Point, whose Quantity depends on one Length determinated by Motion ; so likewise Time may in some measure be looked upon as the Path of an Instant continually gliding along, whose Quantity flows from one Succession, as it were stretched out in Length, and which is demonstrated by the Length of the Space passed over ; and therefore may be best represented by the Flux of a Point or a right Line, which will be often done in what follows.

BUT it must be observed, that we understand by the word Time, that Space of Time wherein Motion is performed ; so that when we treat of philosophical Matters and of Motion, it (*viz.* Time) may be fitly defined with *Aristotle*, *Mensura Motus secundum prius & posterius* ; that is, *the Measure of Motion according to what is past and what is to come* : not indeed as considering the absolute Nature of Time, but that Connection which Motion has with it ; namely, as no Space can be passed over in an Instant by a Body in Motion, but all Motion is performed successively, and according to the Flux of Time, which Motion therefore may be compared with the Quantity of Time, and be measured by its Flux.



LECTURE VI.

Definitions.

I. **M**OTION is a continual and successive Change of Place.

II. CELERITY is an Affection of Motion, whereby a Body in Motion passes over a given Space in a given Time.

III. BUT Rest is the Permanence of any Body in the same Place.

HENCE it follows, that Rest, Motion, and Celerity are twofold, according to the double Distinction of Place, viz. absolute and relative.

IV. ABSOLUTE Motion is the Change of absolute Place, and its Celerity is measured by absolute Space.

V. ABSOLUTE Rest is the Permanence of a Body in the same absolute Place.

VI. RELATIVE Motion is the Change of relative Place, whose Celerity is measured by relative Space.

VII. BUT relative Rest is the Permanence of a Body in the same relative Place.

FROM these it follows, First, That a Person may relatively be at rest, who yet, in respect to absolute Space, may truly and absolutely be in motion: As, for example, if any one should be seated in a Ship, since he keeps the same relative Place, and retains the same Situation and Distance in respect to the other Parts of the Ship, which are looked upon as quiescent, he is relatively at rest; though in the mean while

while he is carried by the same Motion, with the same Celerity, and according to the same Course, as the Ship is borne by the Winds; in which case, all the Parts of the Ship keeping the same Situation amongst themselves, will appear to the Spectator placed within the Ship, as if they were at rest: on the contrary, whilst the Ship is in motion, the Shore and other circumjacent Bodies out of the Ship, will appear to a Spectator in the Ship, to be moved with the same Celerity, but towards contrary Parts, as the Ship, or it will recede from them. The Reason of this Appearance may be easily shewn from Opticks: for those Bodies seem to be at rest, which keep always the same Positions and Distances in respect to the Eye itself; but those Bodies which we see to be moved, we find them change their Distance and Positions in respect to our Eyes. But we will consider this matter a little farther.

SINCE Opticks teach us, that every Body, which is visible, has, by means of the Rays which proceed from that Object, its Image painted on the bottom of the Eye, or Retina; it follows, that those Objects will seem to be moved, whose Images are moved on the Retina; that is, which pass over successively the different Parts of the Retina, whilst the Eye is supposed to be at rest: but those Objects will be looked upon as being at rest, whose Images always occupy the same part of the Retina, that is, when the Motion of those Images are not perceived in the bottom of the Eye. And hence it is, that they who are seated in a Ship, do not perceive the Motion of the Ship: for all the Parts of the Ship being relatively at rest among themselves, keeping the same Position and Distance in respect to the Eye, will have their Images always painted on the same Parts of the Retina; therefore their Motion will not be seen. But when the Spectator turns his Eyes towards the

the Shore, whilst the Ship is moved, it is necessary that any external Object should change its Situation in respect to the Eye, and therefore its Image will successively occupy different Parts of the Retina ; that is, the external Object will seem to move. For the same reason, if the Earth is moved about the Sun, or its own Axis, its Motion will not be perceived by the Inhabitants of the Earth ; *viz.* since the Edifices and all visible Objects on the Earth remaining in the same Parts of the Earth, will always keep the same Position amongst themselves and to the Eye ; but if the Stars, and all other Bodies not fixed to the Earth, are beheld, those, for the same cause as the Shore before, will be seen to move : that is, if the Earth is turned about its Axis from West to East, the Sun and the other Stars will be beheld to move from the East to the West.

BUT leaving for a while the Motion of the Earth, let us return to the Example of a Ship : If the Ship is carried in any Direction whatever, as, for example, towards the East ; and any one sitting in the Prow, should cast a Stone towards the West, with the same Velocity as the Ship itself has towards the East ; the Stone in this case would seem to the Spectator within the Ship, to be moved towards the West, and its relative Velocity would be equal to the absolute Celerity of the Ship itself : yet in truth the Stone would be at rest in the absolute Space, abstracting from the Motion of the Earth, and all that which may arise from Gravity. And if we suppose any Person to be placed out of the Ship hanging in the Air, he would behold the Stone at rest ; but since the Stone is heavy, he would see it move only perpendicularly downwards, not tending more towards the East than towards the West : for the Force impressed on the Stone by the Caster, does nothing else but destroy the equal Force of Motion, that was commu-

communicated to it from the Ship moving in a contrary Direction; for any Body or Space being in motion, likewise all Bodies or Particles of Bodies relatively at rest within that Body or Space, will be moved with the same Celerity, and in the same Direction.

BUT some may object, That the Stone thrown from the Hand of the Caster, lights on the Poup, and gives it a Blow: and since the Stone strikes against the Poup itself, it cannot but be moved. I answer, it is true indeed that those who are within the Ship behold the Stone lighting on and striking against the Poup; but if any one is supposed hanging in the Air out of the Ship, he would not see the Stone moving towards the Poup, but the Poup towards the Stone, and striking it; and the Magnitude of the Stroke, that is received on each Body, will be altogether the same as if the Ship was at rest, and the Stone really impelled towards the Poup with the same Celerity wherewith the Poup arrives at the Stone. For

if there are two Bodies, A and B, however equal or unequal; the Force of Percussion will be the same, whether B with a given Celerity impinges on the Body A at rest, or if B



be at rest, and A with the same Celerity rushes on it: or if each Body should be moved towards the same part, and the subsequent Body A moving the fastest, should impinge on B; the Quantity of the Stroke will be the same, as if B was altogether at rest, and A was only born along with the Difference of Celerities, that is, whereby its Celerity exceeds the Celerity of the Body B: or lastly, if A and B were carried towards different Parts, the Magnitude of the Stroke would be the same, as if one was at rest, and the other moved, with a Celerity that is equal

to the Sum of the former Velocities. I say, in short, the relative Velocity of Bodies always remaining the same, as that wherewith they arrive at one another, the Quantity of Percussion will be likewise the same, however the true Velocities are shared, as will be demonstrated hereafter. But let us return to our Example of the Ship.

IF the Force with which a Stone is thrown by the Caster, is less than that which it receives in this case from the Motion of the Ship, that Stone is truly carried by its absolute Motion towards the same Parts, as the Ship itself: that is, it will be seen to move by the Spectator, whom we have supposed to be placed out of the Ship in the Air, towards the East, with a Celerity, whereby the Celerity of the Ship exceeds the Celerity of the Motion impressed by the Hand of the Caster; but to those placed in the Ship, the Stone will appear to move towards the West, with the very same Celerity that it received from the Hand of the Caster, with which also it will seem to strike on the Poup.

BUT if any one sitting in the Poup should throw a Stone towards the Prow, its true and absolute Motion would be towards the Prow or East, and it would be beheld by our Spectator placed out of the Ship, to be carried with a Celerity that is equal to the Sum of two Celerities, whereof one is that which the Stone receiv'd from the Caster, the other that which was communicated to it from the Motion of the Ship.

ALL these things may be apply'd to the Hypothesis of the Earth's Motion. For if the Earth only revolved about its Axis from the West towards the East, and a Stone or Bullet should be thrown by a Cannon to the West, with that Velocity whereby the Earth is turned about its Axis; the Impetus, that the Bullet receives from the Cannon, would destroy

the contrary Impetus, that was impressed on it by the Earth: so that the Bullet would be at rest in the absolute Space, if we have no regard to the Motion arising from its Gravity. Nevertheless those that live on the Earth, and revolve together with it, would behold the Stone or Bullet swiftly carried towards the West; and if any Wall was oppos'd to its apparent Motion, they would see the Bullet striking against the Wall with the same force as if the Wall was really at rest, and the Bullet impinged against it with the Celerity, which in that case it had received from the Explosion: for, as we said before, the Quantity of the Stroke would be the same, whether the Bullet was thrown with the determinate Celerity against the Wall at rest, or the Wall met the Bullet at rest with the same Celerity.

IF the Force that is impressed on the Bullet by the Explosion of the Gun, be less than that which is communicated to it by the diurnal Motion of the Earth, the Bullet will be really carried towards the East; but because its Velocity is less than that whereby we are revolved towards the East, the Bullet will appear to us to tend towards the West, and any Object that opposes its apparent Motion, will seem to be struck by it with the same force, as if really the Obstacle had remained in the same absolute Space, and the Bullet had impinged on it with the Force it received from the Gun. If, lastly, the Bullet be dislodged towards the East, its absolute Motion will be towards the East, and its Velocity will as much exceed the Velocity wherewith the Earth is moved, as is that which is impressed on the Bullet by the Gun; so that it rushes on and strikes against any Obstacle with that only Difference of Velocity.

AND universally, of all Bodies included in a given Space, their Motions amongst themselves will be the same, their Congress the same, the same Force of

their Percussion, whether that Space is at rest, or moves uniformly in a right Line.

HAVING, and that proluxly enough, explained Motion, Rest, Celerity, as well absolute as relative, we now come to define other Terms.

VIII. *THE Space passed over is the Way that by a Body is run through in its Motion.*

IX. *ITS Length is the right Line that is described by the Center of the moved Body.*

X. *THE Direction of Motion is the right Line whither the moving Body tends.*

XI. *EQUABLE Motion is when the moving Body describes all the Parts of the Length or Space passed over always with the same Celerity.*

XII. *ACCELERATED Motion is that, whose Velocity increases continually.*

XIII. *RETARDED Motion is that, whose Velocity is continually diminished.*

XIV. *MOTION equably accelerated is that to which in equal Times there is always added equal Increments of Velocity.*

XV. *MOTION equably retarded is that, whose Velocity in equal Times equally decreases, even to rest.*

XVI. *A MOMENTUM (which is often called the Quantity of Motion, and also simply Motion) is that Power or Force incident to moving Bodies, whereby they continually tend from their present Places.*

XVII. *BUT an Impediment is that, which obstructs or resists Motion, and destroys, or at least diminishes it.*

XVIII. *THE moving Force is the Power of an Agent to cause Motion.*

XIX. *AN impressed Force is an Action exerted on a Body, to change its State either of Motion or Rest.*

If a Body A at rest is put in motion with a given Celerity, the Force that is impressed on it, and by retaining

receiving which it begins to move with a given Velocity, is called the impressed Force; in which case it only differs from the moving Force in the Manner of conceiving it: for the same Force, as it proceeds from the Agent, is called the moving Force, and as it is received by the Patient, the impressed Force. So likewise if a Body B is moving, a certain determinate Force is required to diminish its Motion, and also a certain determinate Force is necessary to put a thorough stop to its Motion; which, when it is exerted on the Body B, it is called the impressed Force.

I AM not ignorant that there are some Philosophers who do not distinguish the Quantity of Motion from its Celerity; but say those Bodies have equal Motions, that are moved with an equal Celerity, whether the Bodies themselves are equal or unequal, whether one is very small, and the other ever so great, so that each Body is carried with the same Velocity, they contend that the Quantity of Motion in each will always remain the same. But not only Reason, but Experience, shews that Motion is not only increased in the Ratio of the Velocity, but also in that of the Bulk or Magnitude, the Bodies being supposed homogeneous, or of the same Species:

As, for example, let A and B be two Bodies, A the greater, and B the less, and the Momentum or Quantity of Motion of A will be not only greater



than the Momentum of B, if A is moved swifter than B; but if both are carried with an equal Celerity, the Force or Energy whereby the greater Body A is carried, will be greater than that which the Body B has to change its place; because there is required a greater contrary Force of an Obstacle or Impediment to stop the Motion of the greater Body A, than that

which is necessary to destroy the Motion of the less Body B. For if the Body A is an hundred Pounds weight, but the Body B one Pound only, and if in both Bodies the Celerities are equal; the Force that the Body A exerts, and whereby it endeavours to remove an Obstacle, (and therefore the Force of the Impediment resisting and destroying its Motion, will be much greater than the Force of the Motion of the Body B) *viz.* whereby it endeavours to remove the Impediment, and the Force of that Impediment, which is necessarily required to destroy the Motion of the Body B, will be less than the Force of the Impediment that shall be sufficient to take away the Motion of the moving Body A. But we shall hereafter give Theorems, whereby may be estimated the Quantity of Motion, and its Measure be determined.

XX. *THOSE moving Forces are equal, which acting alike, produce equal Quantities of Motion in a given Time.*

XXI. *THOSE Forces are contrary, whose Lines of Direction are contrary.*

XXII. *GRAVITY is a Force bearing downwards, whereby Bodies tend to the Earth in right Lines.*

XXIII. *A CENTRIPETAL Force is that Force whereby a Body continually tends to some Point as a Center; and hence it follows, that Gravity is a certain centripetal Force.*

XXIV. *BUT by a Centrifugal Force we mean a Force whereby any Body is continually sollicit^d to recede from a Center.*

BUT these Forces are always estimated by the contrary Forces, that are able to keep the Bodies in the same state. So if any Body fixed to a String is revolved about an immoveable Center, the Force whereby it endeavours to recede from the Center, is

the centrifugal Force; but the Action of the String resisting and continually drawing back the Body towards the Center, whereby it comes to pass that the Body is always retained in the same Circle, will be as a centripetal Force equal to the centrifugal one, so that one of these Forces may be aptly estimated by the other. So likewise the Force of Gravity of any Body is known by a Force contrary and equal to it, whereby its Descent may be hindered. But that Force may be either the weight of another Body acting contrarily by means of a mechanical Instrument, as, for example, a Balance, or a centrifugal Force that will arise, if that Body revolves in a Circle about the Center of the Earth with some certain and determinate Velocity; or lastly, it may be the Firmness and Resistance of another Body on which the pressing Weight rests.

XXV. *THE Accelerating Quantity of any Force, is the Measure of the Velocity which that Force generates in a given time.*

AT the same Distance from the Earth, all Bodies, of how unequal weight soever they are, do descend equally swift, and therefore their accelerating Forces are equal; but at unequal Distances they are unequally accelerated, viz. at a greater, less; at a less Distance, more.





LECTURE VIII.

HAVING finished the Definitions, which were necessary to explain such things as are not sufficiently clear, or those Terms that are less usual, we now come to treat of the Philosophical Axioms. But since the Object of Natural Philosophy are Bodies and their Actions on one another, which are not so easily and distinctly conceived, as those simple Species of Magnitudes which are the Subject of Geometry; I would not have any one, in physical Matters, insist so much on a rigid Method of Demonstration, as to expect the Principles of Demonstrations, that is, Axioms so clear and evident in themselves, as those that are delivered in the Elements of Geometry: for the Nature of the thing will not admit of such. But we think it sufficient, if we deliver such as we apprehend are congruous to Reason and Experience, whose Truth shines out, as it were, at first view, which procure the Belief of such as are not obstinate, and to which nobody can deny his Assent, unless he professes himself to be altogether a Sceptick.

BUT also in demonstrating, it is necessary to make use of a more lax sort of Reasoning, and to exhibit Propositions that are not absolutely true, but nearly approaching to the Truth. As, for example, when it is demonstrated that all the Vibrations of the same Pendulum made in the small Arches of a Circle, are of equal Duration; it is here supposed, that the small Arch of a Circle and its Chord are of the same Declivity, and of the same Length: which however, if we regard the rigid Truth, is not to be admitted;
but

but in Physicks, this Hypothesis varies so little from the Truth, that the Difference ought justly to be neglected, and the Disagreement of the Vibrations arising from that difference, is altogether insensible, as is proved by Experience. So likewise that eminent Philosopher and Geometer, Dr. Gregory, in his *Elements of Catoptricks and Dioptricks*, makes use of a more lax Geometry, by assuming Lines and Angles as equal, that in reality are unequal, tho' they accede nearly to an Equality: And so he solves many beautiful Physical Problems, which otherwise would prove very intricate. And also this Method seems to be approved of sometimes by Sir Isaac Newton himself, as may be seen in *Prop. 3. Lib. 2.* of his *Philosophiæ Naturalis Princip. Math.*

BUT if there are any who harden their Minds against such Principles and Demonstrations, and will not suffer themselves to be convinced by Propositions sufficiently manifest; we leave such to enjoy their supine Ignorance, nor do we think them worthy to be admitted to the knowledge of the true Philosophy.

A X I O M S.

- I. THERE are no Properties or Affections of a Non-entity or Nothing.
- II. NO Body can be naturally annihilated.
- III. EVERY Mutation induced in a natural Body, proceeds from an external Agent; for every Body is a listless Heap of Matter, and it cannot induce any Mutation in itself.
- IV. EFFECTS are proportionable to their adequate Causes.
- V. THE Causes of Natural Things are such, as are the most simple, and are sufficient to explain the Phænomena: for Nature always proceeds in the simplest and most expeditious Method; because by this Manner of operating the Divine Wisdom displays itself the more.

VI. NATURAL Effects of the same kind have the same Causes: as the Descent of a Stone and a Piece of Wood proceeds from the same Cause; and there is also the same Cause of Light and Heat in the Sun and in a Kitchen-Fire, of the Reflection of Light in the Earth and in the Planets.

VII. IF two things are so connected together, that they perpetually accompany each other, that is, if one of them is changed or removed, the other likewise will be in the same manner changed or removed; either one of these is the Cause of the other, or they both proceed from the same common Cause.

So if there be a magnetick Needle moveable on an Axis, upon a Loadstone being brought near it, and moved round it, the Needle will immediately be moved with the same Tenour: and if the Motion of the Loadstone be stopped, the Circulation of the Needle likewise stops; and it will again begin to revolve, as soon as the Loadstone is again moved: whence nobody can doubt, but that the circular Motion of the Needle depends on the Motion of the Loadstone. So likewise since the Flux and Reflux of the Sea happens at the same place when the Moon comes to the same Hour-Circle, and constantly observes its Motion; insomuch that a Period of the Tides so precisely answers to the Period of the Moon's Motions, that there has been found no Aberration for so many Ages; for it is retarded 48 Minutes every day, and at the Conjunction of the Sun and Moon, the Tides are always at the highest, and in their Quadratures at the lowest: Whence it must be granted, that the Flux of the Sea depends on the Motion of the Moon, and its Situation in respect to the Sun.

VIII. *ANY Body being moved in any Direction, all its Particles which are relatively at rest in it, proceed together in the same Direction with the same Velocity; that is, a relative Place being moved, that which is placed therein will be also moved.*

IX. *EQUAL Quantities of Matter carried along with the same Velocity, their Momenta or Quantities of Motion will be equal.*

For the Momentum of any Body is the Sum of the Momenta of all the Particles composing that Body; and therefore where the Magnitudes and Numbers of the Particles are equal, the Momenta will be equal.

X. *EQUAL and contrary Forces acting on the same Body, destroy their mutual Effects.*

XI. *BUT from unequal and contrary Forces there is produced a Motion Equivalent to the Excess of the greater Force.*

XII. *A MOTION produced from conspiring Forces, that is, acting in the same Direction, is equivalent to their Sum.*

XIII. *IF what is equivalent be either augmented, or its contrary diminished, then it becomes the greater.*

THEY who would philosophize mechanically, exhibit the two following.

XIV. *ALL Matter is every where of the same Nature, and has the same essential Attributes, whether it is in the Heavens or on the Earth, whether it appears under the form of a fluid Body, or a hard, or of any other whatever; that is, the Matter of any Body, for example of Wood, does not differ essentially from the Matter of any other Body whatever.*

XV. *BUT the different Forms of Bodies are nothing but the different Modifications of the same Matter; and depend on the various Magnitude, Figure, Texture, Position,*

sition, and other Modes of the Particles composing Bodies.

XVI. So likewise the Qualities, or Actions or Powers of some Bodies on other Bodies, arise only from the former Affections and Motion conjointly.

BUT the Philosophers suppose Matter to be the common Subject or *Substratum* of all Forms and Qualities; that it is indifferent to them all, since it is capable of them all, and remains the same under whatever Form it may appear; and whence it is called by the Peripateticks *Materia Prima*, or the first Matter.

BUT although Forms and Qualities are altogether accidental to Matter, yet they necessarily and essentially belong to Body, which is made up of Form and Matter together: as, for example, although the Matter of Wood is altogether indifferent to this or that Form or Figure and Texture of Particles, which being varied infinite ways, remains the same; yet the Wood cannot subsist without that determinate Modification of Particles, which constitutes the Form of a wooden Body; which being removed, the Wood perishes, and the same Matter passes into a Body of another kind. But that the Form of a wooden Body consists in the Modification of Particles, is manifest upon putting the Wood into the Fire, whereby its Matter is deprived of its Form; for by the Force of the Fire is dissolved the Connection and Texture of Particles, and some part of them passes into Smoke and Vapours, and the other is reduced into Ashes.

MANY Examples are alledged by the Philosophers, to shew that the various Magnitudes, Figures, and Textures of the Particles of the same Matter may produce various Forms of Bodies, and that various Qualities may arise from their various Motion and Position; of which we shall here give some Instances.

FIRST

FIRST of all, when the Particles of Water are rarefied by the Heat of the Sun, they are rais'd out of the Sea aloft into the Air under the form of Vapours; but this new Form is owing to nothing but the changed Situation of the Parts: for by Rarefaction it happens, that the Particles of Water contain in themselves more, and perhaps larger Spaces, either altogether empty, or only replete with a very rare *Æther*; whence their Matter takes up a larger Space than an equal Quantity of aerial Matter, and it is rendered less intensively dense than the Air, and therefore it is forced upwards after the same manner as Cork is, when it is immersed in Water: Nor do these Vapours ever stop, till they arrive at Air of the same Gravity, where they rest relatively, and compose Clouds that are of a thousand Shapes.

Now when by the Course of the Winds the Air is rendered less heavy, the Vapours retaining the same Gravity, must necessarily subside, and in their fall being condensed by the Resistance of the Air, and driven into a less space, they lose their Form, and falling to the Earth, assume the Species of Rain.

MUCH the greatest part of this is carried to the Sea in Rivers, to be changed again into Vapours; but some part of it mixes with the Earth, and being there deposited, enters into the Roots and Seeds of Trees and Herbs, whence it arises into other and new Species of Bodies. And the same Rain-Water composes different Bodies, as it enters into the different Seeds of things; some passes into Plantanes, some into Grasses, others into Flowers, others into Oaks, Ash, Beech, and other innumerable sorts of Trees and Plants.

NOR in the same Plants does the Rain-Water remain altogether similar, since all Plants consist of innumerable heterogeneous Parts: so in Hemp, for example, the Root is of one form, the Stalk of another,

ther, the slender Fibres of another, the Flowers of another, the Seed of another, the Cells containing the Seed of another.

THE Structure of Vessels in the same Hemp is also very various; (for, as in an animated Body, every Plant has its Vessels serving for the Circulation of the Humours) and these Parts are of very different Properties: the Stalk, for example, is a ligneous Body, and after being dried, becomes very friable; whilst the Rind, or the Membrane covering the Stalk, consists of oblong, very slender and folding Fibres variously connected together.

THE Hemp-dressers separate this Membrane from its Stalk, and after they have handled it a thousand different ways, they spin its Fibres into oblong Threds; and the Position and Situation of the Particles being changed, these Fibres take another and a very different Form from that which they had in the green Plant.

THEN these Threds being wound up, their smallest Particles continuing the same, appear in the shapes of Bottoms. The Weavers variously connect and weave these Threds, and by their Art make out of them Webs, that yield Garments for Men. These, lastly, being reduced to Linnen Rags, are soaked in Water, and by wooden Hammers reduced, as it were, into a soft Pulp; which at length, the aqueous Moisture being dried up, is transmuted into Paper, which if it be put into the Fire, is turned partly into a very fine Powder, and partly vanishes in Smoke.

BUT all these multifarious Forms under which the same Matter appears, arise from nothing but the changed Figure, Magnitude, and Texture of Particles, and only depend on them.

So when Metals are melted, the Coherence of their Parts is dissolved by the Force of the Fire, and the

the metallick Particles separated from one another, are hurried about with a most rapid Motion, whence they assume the Form of a fluid Body.

HENCE also (as it seems) arises the Solution of Salts and Metals in Menstruums: for their Parts are separated from one another by Fermentation, and being resolved into their least Particles, they are agitated by the Motion of the Fluid itself, whence they will appear as fluid Bodies. From these Figures, and the other Modifications of Bodies and their Parts, there arise many Effects, many Qualities proper to each sort of Bodies, which must necessarily be destroyed if the Constitution of Parts is changed. So of the same Matter, as, for example, Iron, may be formed Keys, Plow-shares, Files, Saws, and innumerable other Instruments accommodated to various Uses, whose Qualities and Effects solely depend on their Figures: for whence have Keys the power of opening Doors, but from their Figure, Magnitude, and Congruity of Parts with the Parts of the Locks into which they are put? Whence have Wedges and Plow-shares their power of splitting and dividing Bodies? Have not the Writers of Mechanicks demonstrated, that they derive it from their Figure alone? Whence are those so regular Motions performed in Clocks or Watches, but from the Wheels being disposed amongst themselves, adapted to one another and joined together? Whence, lastly, is it, that such great Effects are produced by artificial Machines? Certainly the Reason is to be deduced only from their Fabrick.

NOR are natural Bodies less indebted to the Constitution and Modification of their Parts than artificial ones; for all their Operations proceed only from the Motion, Situation, Order, Figure, and Position of their Corpuscles; which being changed in any Body, the Qualities of that Body are also actually changed.

IF the Superficies of a Body is uneven and rough, it reflects every way the Light incident on it; because the superficial Parts receiving and remitting the Light, are not all in one and the same regular Superficies, but are placed in almost infinite, and those different Planes; whence the Light falling on these various Planes, must necessarily be reflected every way. Hence Ice, which when it is whole and smooth is almost of no colour, yet broken into pieces, and having rough and angular Parts, it appears white, *viz.* when it reflects Light copiously and on all sides. The Reason is also the same for Water's growing white, upon its being turned into Froth.

BUT such is the Structure of most visible Bodies, that their Surfaces suffocate part of the Rays that fall upon them, and remit part. If their Surfaces are so constituted, as to reflect or suffocate equally all sorts of Rays, their Colour would be white or black, or brownish, betwixt black and white; for the white Colour does no other ways differ from black, than as white Bodies reflect abundance of Rays of all sorts, but the black only a few. This appears from the Shade of an opaque Body, that whilst the Sun shines, is cast on a white Wall; for that part of the Wall where the Shadow is, as it receives much fewer Rays than the other Parts, it also reflects much fewer Rays, so that in respect to the other Parts it appears black. But if those other Parts of the Wall had not received more Rays than that Part where the Shadow fell, it would be all of the same Colour, *viz.* white.

IF the Texture of the Superficies is such, that it reflects one sort of Rays more plentifully, and all the rest more sparingly, the Colour of the Superficies will approach to that, which arises from the Rays that are reflected in greater number. This may be hence demonstrated, that the Colour of the same

Object will be different, as it receives different sorts of Rays, the rest being intercepted; which was first discovered by the Great Sir *Isaac Newton*. So if through a Glass Prism red Rays (for so we call those that produce a red Colour) are cast on a blue Object, the Object will change its blue Colour, and put on a red one; but if it receives only yellow Rays, then its Colour will be changed into yellow: if the incident Rays are blue, it will appear blue; and that Colour will be more vivid than all the other Colours, because it reflects many more of these Rays, and suffocates fewer of them than of the rest.

If the Superficies of a Body is exactly smooth, that is, without any Roughness or Unevenness; and it reflects from it Rays in great plenty, it will reflect the Rays that proceed from any Object in such a manner as to afford a visible Image of that Object. And for that reason, the Bodies having such Surfaces are called *Speculums*. If the Speculum is a Plane, the Image will be equal to the Object itself, and its place will be behind the Speculum at a Distance equal to that of the Object itself from the Speculum; if the Speculum be a spherical Concave, and the Radiant Object is farther distant from it than the fourth part of the Diameter of the Sphere, the Image will appear hanging in the Air between the Object and the Speculum, and will be less than the Object; if the Object be placed in the Center, the Image will be also there, and equal to it; if the Object proceeds beyond the Center towards the Speculum, so that its Distance from it shall be a fourth part of the Diameter of the Sphere, the Image from the Speculum will pass beyond the Center, and be greater than the Object; but when the Object shall have arrived at the Distance of one fourth of the Diameter of the Sphere, then the Distance of the Image will become infinite; but if it

H approaches

approaches but a very little nearer to the Speculum, the Image will be behind the Speculum, and greater than the Object itself. All these Phenomena that are so very different, proceed only from the Alteration of Distances, every thing besides remaining the same.

LET us now consider those various and altogether contrary Effects, which arise from an Alteration only in Situation and Position, every thing else being in the same State, except such things as depend on the Mutation of the Situation.

ALL Philosophers now acknowledge the Sun to be at rest in the Center of his System, but the Earth, like the other Planets, to be carried about him in the space of a Year; yet the Earth moves about the Sun in such a manner, that its Axis remains always parallel to itself, not indeed perpendicular to the Plane of the Earth's Orbit, but inclined to it in an Angle of $66^{\circ}.30'$. And by reason of this Parallelism and Inclination, it follows that the Earth must sometimes turn one of his Poles towards the Sun, sometimes the other; and therefore all the Parts of the Earth will have different Situations, in respect of the Sun. From this Change in its Situation, arise all those Vicissitudes of Season that yearly happen, *viz.* Summer, Winter, Spring, and Autumn: for if the Axis of the Earth were perpendicular to the Plane of its Orbit, there would be no Alterations of Seasons, no Differences in the Days and Nights, but every Part of the Earth would always receive, after the same manner, the equal Forces of the Sun's Rays.

BUT since every part of the Earth changes its Situation in respect of the Sun, and receives its Rays now more, at another time less oblique, now for a shorter duration, then for a longer; there will thence arise different and altogether contrary Appearances. As in the Autumn the Corn becomes ripe, and the Fruits mellow; yet the Fields by degrees

Lect. 8. *Natural Philosophy.* 99

greens lose their green and pleasant Colour, and the Leaves fall from the Trees. Afterwards the Winter approaching, all things become cold and horrid, the Snow covers the highest Mountains, and the Trees labour with its weight; and what is wonderful, the Waters of the Sea become stable and firm, and what before was only passable by Ships, now bears Tents and Armies.

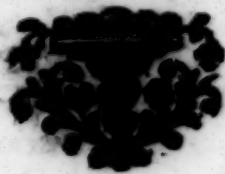
BUT the Earth continually revolving in its Orb, each Part thereof changes its Situation in respect of the Sun, and what before was turned from, begins now to respect the Sun: and in the mean while the Snows disappear, the Fields are clothed with Grass, and the Trees adorned with Leaves, the Horse now no longer abides in the Stable, nor the Plowman at the Fire-side; but a new and joyful Face of things appears, and the Year returns through the Summer to the Autumn.

Now since so many different, so many contrary Effects happen from the sole Alteration of the Situation, and so various Phenomena follow from it, all the other Causes remaining the same; certainly from the Position, Distance, Magnitude, Figure and Structure of the Parts composing Bodies, from the Motion and Subtlety of Effluvia, from the Congruity of Bodies, and their Respect to other Bodies; I say, from all these joined and combined together variously, and almost after infinite manners, may proceed almost infinite different Forms, Affections, and Operations amongst themselves of Bodies, nor is any thing to be found in Nature, that does not depend on these. For if these are changed, the Forms, Qualities, and Operations of Bodies will be also changed. For Example, it is certain that the magnetick Attractions and Directions arise from the Structure of Parts; for if a Loadstone be struck hard enough, so that the Position of its internal

Parts be changed, the Pole of that Loadstone will also be thereby changed. And if a Loadstone be put into the Fire, insomuch that the internal Structure of its Parts be changed or wholly destroyed, then it will lose all its former Virtue, and will scarce differ from other Stones.

BUT although it is in the general shewn, that the magnetick Operations do in some sort proceed from the internal Constitution of Parts, yet the Modus of Operation, deduced from mechanical Principles, and such as are easy to be understood, is not yet discovered. And what some generally boast of, concerning Effluvia, a subtile Matter, Particles adapted to the Pores of the Loadstone, &c. does not in the least lead us to a clear and distinct Explication of these Operations: but notwithstanding all these things, the magnetick Virtues must be still reckoned amongst the occult Qualities.

FROM what has been said, it follows, that the Qualities of Bodies which do not depend on their Forms, and which, the Quantity of Matter remaining the same, are not increased or remitted, but are in all sorts of Bodies upon which we can make any Experiments; are the universal Qualities of all Bodies. For since they do not proceed from the Form or Modifications of Bodies, they must depend on the Matter itself; but since the Nature of all Matter is the same, and any part of it differs from another only by some Modes, the Qualities not produced by these Modes will be the same in all Matter.



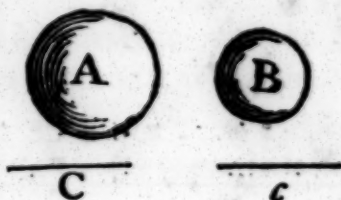
LECTURE IX.

*Theorems of the Quantity of Motion,
and of Spaces passed over by Bodies
in Motion.*

THEOR. I.

N comparing the Motions of Bodies, if the Quantities of Matter of Bodies in motion are equal, the Momenta or Quantities of Motion will be as the Velocities.

LET A and B be two Bodies in motion, having equal Quantities of Matter, and let A be moved with the Celerity C, but B with the Celerity c ; I say, the Momentum or Quantity of Motion in the moveable Body A, is to the Momentum or Quantity of Motion in the moveable Body B, as the Celerity C to the Celerity c ; for if any Force is impressed on the Body A to move it with



a given Velocity C, there is required a double Force to move the Body B with a double Velocity, and a triple Force must be used to move it with a triple Velocity, and only half that Force is necessary to move B with half the Velocity, and so of other Multiples and Submultiples. That is, since [by the fourth Axiom] Effects are proportionable to their adequate Causes, if the Force that is used to move

the Body B, is double that which is applied to move A, the Momentum of that will be also double of the Momentum of this; if the Force is triple, the Motion of the Body B will be likewise triple the Motion of the Body A; if but half the Force is impressed on the Body B, its Momentum will be half the Momentum of A: that is, since the Velocity of the Body A is universally to the Velocity of B, as the Force impressed on the Body A to the Force impressed on B; and as the Force impressed on the moveable Body A to the Force impressed on the Body B, so is the Momentum or Quantity of Motion in A, to the Momentum or Quantity of Motion in B, the Velocity of the Body in motion A will be to the Velocity of the Body in motion B, as the Motion of A to the Motion of the moveable Body B. *Q. E. D.*

Cor. If the Momenta are as the Velocities, the Quantities of Matter in the moveable Bodies will be equal,

THEOR. II.

IN comparing Motions, if the Celerities are equal, the Momenta or Quantities of Motion of Bodies will be as the Quantities of Matter in those Bodies; or if the moveable Bodies are homogeneous, as their Magnitudes.

LET A and B be two Bodies in motion, and each of them carried with the same Celerity C; I say the Momentum of the Body A, is to the Momentum of the Body B, as the Quantity of Matter in A to the Quantity of Matter in B. For if the Quantity of Matter in A is double that in B, A may be divided into two parts, each



C—

whereof, will have as much Matter, and consequently [by Axiom 9.] as much Motion as B has:

viz.

viz. since each Body is moved with the same Velocity, so that the Momentum of the Body A will be double the Momentum of the Body B. If the Quantity of Matter in A is triple of that in B, A may be divided into three parts, each whereof will have a Quantity of Motion equal to that which is in B; and universally whatever proportion the Matter in A has to the Matter in B, the same proportion will have the Momentum of A to the Momentum of B, if each Body is carried along with the same Velocity.

If the Bodies are homogeneous, their Quantities of Matter will be as their Magnitudes or Bulks; and consequently their Motion will be also in the same Ratio of their Magnitudes.

Cor. If the Momenta are as the Quantities of Matter, the Celerities of the Bodies will be equal.

THEOR. III.

In comparing the Motions of any Bodies, the Ratio of the Momenta is composed of the Ratios of the Quantities of Matter, and of the Celerities.

Let A and B be any two moveable Bodies, and let A be moved with the Celerity C, but B with the Celerity c; I say, the Momentum of A is to the Momentum of B, in a

Ratio compounded of the Ratio of the Quantity of Matter in A, to the Quantity of Matter in B, and of the Ratio of the

$$\frac{(A)}{C} \quad (G) \quad \frac{(B)}{c}$$

Celerity of the Body A, to the Celerity of the Body B. Let there be a third Body G, whose Matter is equal to that which is in A, but let it be moved with the Celerity of the Body B. From the Elements the Ratio of the Momentum of the Body A, to the Momentum of the Body B, is compounded of the Ratio of the Momentum of the Body A to the

Momentum of the Body G, and of the Ratio of the Momentum of the Body G to the Momentum of the Body B: but [by Theor. 1.] the Momentum of the Body A is to the Momentum of the Body G, as the Celerity C is to the Celerity c ; and since G and B are moved with the same Celerity, the Momentum of the Body G will be to the Momentum of the Body B, as the Quantity of Matter in G or A to the Quantity of Matter in B. And therefore the Momentum of the Body A will be to the Momentum of the Body B, in a Ratio compounded of the Celerity C to the Celerity c , and of the Quantity of Matter in A or G, to the Quantity of Matter in B. *Q. E. D.*

Cor. 1. IF the Bodies are homogeneous, the Ratio of the Momenta will be compounded of the Ratio of the Magnitudes and of the Celerities.

Cor. 2. IF it be made as A to B, that is, as the Quantity of Matter in A to the Quantity of Matter in B, so is the right Line D to the right Line E; and let be compleated the Rectangles under D and C,



and under E and c , the Momentum of the moveable Body A will be to the Momentum of the moveable Body B, as the Rectangle DC to the Rectangle E c .

FOR because it is as A to B, so D to E, the Ratio compounded of the Ratios of A to B and of C to c , will be equal to the Ratio compounded of the Ratios D to E and of C to c ; but [by 23 *El.* 6.] the Ratio compounded of the Ratios of D to E and of C to c , is equal to the Ratio of the Rectangle DC to the Rectangle E c , and [by this third Theor.] the Ratio of the Momentum of the moveable Body A to the Momentum of the moveable Body B, is equal to the Ratio compounded of the Ratios of A to B, or of D to E, and of C to c ; wherefore it will be as the Rectangle DC to the Rectangle

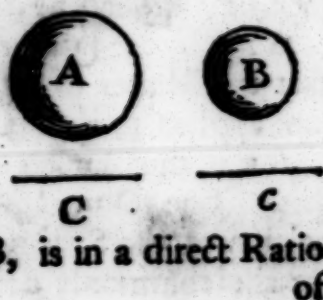
Rectangle $E c$, so is the Momentum of the moveable Body A to the Momentum of the moveable Body B. The Momentum therefore of any Body may be considered as a Rectangle produced from the Multiplication of the Bulk or Quantity of Matter contained in that Body into its Celerity.

Cor. 3. WHEREFORE whatever is demonstrated concerning the Proportion of these Rectangles, the same will be also true concerning the Momenta of Bodies, their Momenta being proportionable to these Rectangles. For example, if it be as D to E, or as A to B, so is c to C, in this case the Momenta of the moveable Bodies will be equal: for rectangular Parallelograms having their Sides reciprocally proportionable, are [by 14 *El.* 6.] equal; and on the contrary, if the Rectangles are equal, their Sides will be reciprocally proportionable: that is, if the Quantities of Matter, or in Bodies of the same kind, their Magnitudes are reciprocally proportionable to their Celerities, their Momenta will be equal; and conversely, if the Momenta are equal, it will be as the Quantity of Matter in one to the Quantity of Matter in the other, so reciprocally the Celerity of this to the Celerity of that: hence also may be demonstrated the following

THEOR. IV.

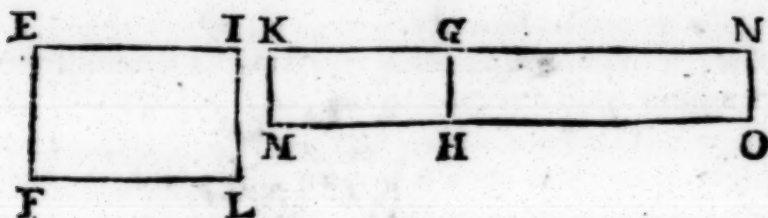
IN compared Motions, the Ratio of the Celerities is compounded of the direct Ratio of the Moment, and of the reciprocal Ratio of the Quantities of Matter.

LET A and B be two moveable Bodies, and let A be carried along with the Celerity C, but B with the Celerity c : I say, C is to c , that is, the Celerity of one of the Bodies A to the Celerity of the other B, is in a direct Ratio



of

of the Momentum of the Body A to the Momentum of the Body B, and a reciprocal Ratio of the Matter in A to the Matter in B, that is, directly as the Matter in B to the Matter in A. Let it be as A to



B, so the right Line EI to the right Line KG; and let IL be equal to C, but GH equal to c ; and let be compleated the Rectangles EL, KH. By what has been said above, the Rectangles EL, KH, will represent the Momenta of the moving Bodies A and B respectively: to GH let be applied the Rectangle HN equal to the Rectangle EL. Since therefore HN is equal to EL, it will be [by 16 *El.* 6.] as IL to GH, so is GN to EI; but the Ratio of GN to EI is equal to the Ratio of GN to GK, and of GK to EI; that is, equal to the Ratios of the Rectangle HN or EL to the Rectangle KH, and of GK to EI. Wherefore the Celerity C or IL will be to the Celerity c or GH, in a Ratio compounded of the Ratio of the Momentum EL to the Momentum KH, and of the Matter GK to the Matter EI: that is, the Velocity of any Body is always as its Momentum applied to its Matter. *Q. E. D.*

AND by the like manner of Reasoning it is found, that the Matter of any Body is always as the Momentum applied to its Velocity.

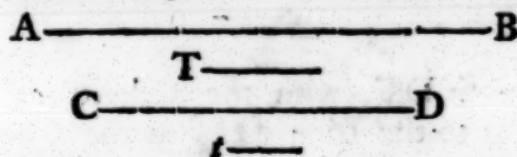
AND thus much concerning the Momenta of Bodies. The following Theorems concerning the Proportion of the Spaces passed over by Bodies in motion, are also commonly demonstrated.

THEOR.

THEOR. V.

IN compared Motions, if the Celerities of the moving Bodies are equal, the Spaces run over by them, will be directly as the Times wherein the Motions are performed.

LET a Body in motion run the length A B, in the Time T, with an equable and uniform Motion; also



let the same or another Body in motion, carried along with the same Velocity, run the length C D, in the Time: I say, the Line A B is to the Line C D, as the Time T to the Time t . For if the Time T is double that of t , it may be divided into two Parts, each whereof will be equal to t ; so that each Space run over with the same Velocity in these equal Parts of Time, will be equal to the Space run over in the Time t , and the two Spaces taken together will be double the Space run over in the Time t . In the same manner, if T is triple of t , T may be divided into three equal Parts, and the Spaces run over in each of these Times, will be equal to the Space run over in the Time t ; and therefore the three Spaces taken together, will be triple the Space run over in the Time t . The same may be shewn of other Multiples and Submultiples: wherefore universally, whatever proportion T has to t , the same will the Space run over A B have to the Space run over C D. Q. E. D.

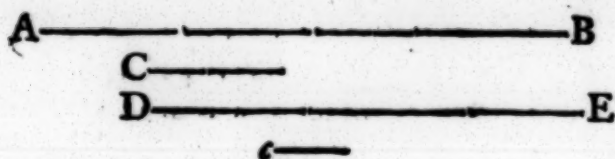
Cor. If the Times are as the Spaces run over, the Celerities will be equal.

THEOR. VI.

IN compared Motions, if the Times of the Motions are equal, the Spaces run over will be as the Celerities.

LET

LET any moving Body run in a given Time the length AB , with the Celerity C ; and in the same, or an equal Time, let the same or another moving Body



run the length DE , with the Celerity c ; I say the Line AB is to the Line DE , as the Celerity C to the Celerity c . For if the Celerity C is double of c , the Space AB run over with the Celerity C , will be double the Space DE run over with the Celerity c ; if the Celerity C is triple of c , the Length AB will be also triple the Length DE ; if C is the half of c , AB will be the half of DE : and universally, since equal Times are spent in running over the Lines, whatever proportion the Celerity C has to the Celerity c , the same will the Length run over AB have to the Length run over DE . *Q. E. D.*

Cor. IF the Celerities are as the Spaces run over, the Times will be equal.

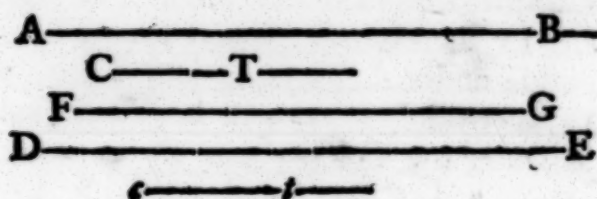
THE two first Theorems, as also the fifth and this sixth, might be demonstrated universally by Equimultiples, after the manner of *Euclid*; but since they are so evident, that they may be looked upon as Axioms, they scarce need so great an Apparatus of Demonstration.

THEOR. VII.

THE Lengths run over are in a Ratio compounded of the Ratios of the Times and of the Celerities.

LET the Line AB be run over with the Celerity C , in the Time T ; and the Line DE with the Celerity c , in the Time t : I say, the Ratio of AB to DE

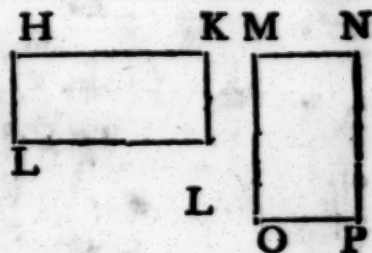
DE is compounded of the Ratio of the Celerity C to the Celerity c , and the Ratio of the Time T to the Time t . Let the Line FG be run over in the



Time T, with the Celerity c ; it is manifest, that AB is to DE in a Ratio compounded of the Ratios of AB to FG and of FG to DE. But because AB and FG are run over in the same time, it will be as AB to FG, so the Celerity C to the Celerity c ; but since the moving Bodies describe the Lines FG and DE with the same Celerity, it will be [by Theor. 6.] FG to DE as the Time T to the Time t . Wherefore since the Ratio of AB to DE is compounded of the Ratios of AB to FG, and of FG to DE, it will likewise be compounded of the Ratios that are equal to these Ratios, *viz.* of the Ratio of the Celerity C to the Celerity c , and of the Time T to the Time t .

Cor. 1. If HK be made equal to C, HI equal to T, also MN equal to c , and MO equal to t , and let be compleated the

rectangular Parallelograms HL, MP; it will be AB to DE, as the Rectangle HL to the Rectangle MP: for [by 23 *El.* 6.] the Rectangle HL is to the Rectangle MP, in a Ratio compounded of the



Ratios of HK to MN, and of HI to MO; but [by the preceding Theorem] the Space run over AB is

is to the Space run over D E, in a Ratio compounded of these Ratios: whence these Spaces run over may be considered as Rectangles produced from the Times multiplied into the Celerities.

Cor. 2. If therefore the Spaces run over are equal, the Rectangle under the Celerity and Time, wherein one Space is passed over, will be also equal to the Rectangle under the Celerity and Time, wherein the other Space is passed over; and consequently it will be as the Celerity to the Celerity, so reciprocally is the Time to the Time, [by 14 *El.* 6.] that is, if the Spaces run over are equal, the Times will be reciprocally as the Celerities.

T H E O R. VIII.

IN compared Motions, the Ratio of the Times is compounded of the direct Ratio of the Length, and of the reciprocal one of the Celerities.

THIS Theorem may be demonstrated from the former, after the same manner as the fourth follows the third: but for the sake of Perspicuity, we shall thus briefly evince it.

A ————— B	Let the Length A B be run over in the Time T, with the Celerity C; also let the Length D E be run over in the Time
K ——— T ——— C ———	
D ————— E	
t ——— c ———	

t , with the Celerity c ; I say, the Time T is to the Time t in a Ratio compounded of the direct Ratio of the Length A B to the Length D E, and of the reciprocal Ratio of the Celerity C to the Celerity c . Let K be the Time wherein the Length A B may be run over with the Celerity c , the Ratio of the Time T to the Time t , will be compounded of the Ratio of T to K, and of K to t ; but [by the Corol. of the preceding Prop.] it is as T to K, so c to C (since the

the ſame Space is run over in each of the Times) and as K to t , ſo is (by the Cor. of Theor. 5.) the Length AB to the Length DE . Wherefore T will be to t in a Ratio compounded of the Celerity c , to the Celerity C , and of the Length AB to the Length DE ; that is, the Times are in a Ratio compounded of the reciprocal Ratio of the Celerities, and the direct one of the Lengths. *Q. E. D.*

AFTER the ſame manner may be ſhewn, that the Celerities are in a direct Ratio of the Lengths, and a reciprocal one of the Times.

Cor. 1. AND hence it follows, that the Time is as the Space run over applied to the Celerity.

Cor. 2. The Celerity likewise is as the Space run over applied to the Time.

THE third and ſeventh Theorem may be demonſtrated from this univerſal Theorem, *viz.*

IF any Effects depend at the ſame time on ſeveral Cauſes, in ſuch a manner, that as they are augmented or diminished in the ſame Ratio wherein any of the Cauſes are augmented or diminished, thoſe Effects will be in a Ratio compounded of all the Cauſes: that is, if the Cauſes A, B, C , acting together do produce the Effect E , which, all the reſt remaining the ſame, is as ſome of the Cauſes; and if other Cauſes a, b, c , reſpectively like the former, and acting after the ſame manner, do produce the Effect e ; it will be as E to e , ſo $A \times B \times C$ to $a \times b \times c$. Which may be eaſily ſhewn, by almoſt the ſame Method that we uſed in the preceding Demonſtrations.

AFTER the like manner, if the ſame Effect depends on ſeveral things at once, whereof ſome aſſiſt or augment the ſame, in that Ratio whereby they are augmented; but others impede or diminish it, in the ſame Ratio whereby they are augmented: the Effect will always be directly as the aſſiſting Cauſes, and reciprocally as the impeding or diminishing Agents.

THE

THE seventh Theorem is thus demonstrated in the Style of Sir Isaac Newton:

THE Celerity being given, the Space run over is as the Time; and the Time being given, the Space run over is as the Celerity: wherefore neither of them being given, it is as the Celerity and Time conjointly.

So likewise may the eighth Theorem be shewn.

THE Celerity being given, the Time is directly as the Space run over; and the Space being given, the Time is reciprocally as the Celerity: wherefore neither of them being given, the Time will be directly as the Space and reciprocally as the Celerity.

IN like manner may the third and fourth Theorem be explained, and we shall sometimes make use of this Method for the sake of Brevity.

LECTURE X.

IN the Demonstrations of the preceding Lecture we have shewn a Method, whereby Physical Matters may be first reduced to Geometry, and afterwards to Arithmetick: for since it is there demonstrated, that the Motions of Bodies are as Rectangles under their Celerity and Quantity of Matter; from the Celerity and Quantity of Matter of any Body being given, there will be given its Momentum, viz. equal to a *Factum* or Product made of the Celerity of the Body into its Quantity of Matter. As, for example, let the Body A be 8 parts, but B be 6 parts, the Celerity of A as 5, and the Celerity of the Body B as 3; the Motion of

of the Body A will be 40, and the Motion of the Body B but 18 Parts.

So from the given Momentum and Quantity of Matter of any Body, may be known its Celerity; viz. if its Momentum be divided by its Quantity of Matter, the Quotient will give its Velocity. For let the Motion in the Body A be 40 parts, and its Quantity of Matter 8 parts; let also the Motion in the Body B be 18 parts, and its Quantity of Matter 6 parts: divide 40 by 8, the Quotient will give 5, viz. the Velocity of the moving Body A; and by dividing 18 by 6, the Quotient will give 3, the Velocity of the moving Body B.

SINCE by Examples things become more plain, and Numbers are to be made use of in Practice, that the young Beginners may accustom themselves the better to these, let us illustrate the Science of Motion by Numbers, and make use of both specious and numerical Arithmetick; for from specious Arithmetick are discovered general Canons, which afterwards may be applied to particular Numbers.

So let A denote the Quantity of Matter in any given Body B, but C stand for its Celerity, and let its Momentum be called M; or rather let these Letters be put for the Numbers that are proportionable to those Quantities: then it will be $C \times A = M$ and $C = \frac{M}{A}$ and $A = \frac{M}{C}$.

IN like manner, since the Space run over is always proportionable to the Rectangle under the Celerity and the Time; if the Space be called S, the Time T, and Celerity C, it will be $S = C \times T$; and $C = \frac{S}{T}$; and $T = \frac{S}{C}$; and therefore since it is

$M = A \times C$, it will also be $M = \frac{A \times S}{T}$: or if T is given,

it will be $M = A \times S$; that is, the Momentum of any Body is as its Matter drawn into the Space run over by it in a given Time. Many other Conclusions likewise, which some deliver as the Laws of Motion, may be deduced from what has hitherto been demonstrated; but since any Tyro may easily of himself discover them all, it is not worth while to produce them here.

FROM what has been demonstrated, it is manifest, that the Momentum of any Body arises from the Motion of all its Parts: for in each Particle of the Body there is an Impetus or Force of moving, and from the Sum of these Forces is compounded the Impetus or Quantity of Motion of the whole Body.

HENCE also may be gathered, that by how much the greater is the Quantity of Matter in Bodies, by so much the greater ought the Force to be, in order to move those Bodies with a given Velocity, and therefore their Momenta will be greater in the same Ratio. If therefore there are two Bodies carried along with the same Velocity, the Quantities of Matter in them will be always as their Momenta; so that if Bodies which are equal in Bulk, and moved with the same Velocity, have unequal Momenta, it is necessary that the Quantities of Matter in them are unequal: and that which has the less Momentum, will have more Pores or Spaces either altogether empty, or replete with some Matter which does not partake of the Motion of the whole Body, whose Pores it is supposed to fill. So, for example, if there are two Globes, one of Cork, the other of Lead, both of the same Magnitude, and moved with the same Velocity, since it is known by Experience, that the Momentum of one is much greater than the Momentum of the other, it must follow that there are many more Pores in one than in the other; which
it

it must be allowed, are either altogether empty, or replete with some most subtile Matter, which may so freely pass through its Pores, as not to partake of the Motion of the Body whose Pores it possesses.

BUT that this Matter may pervade the Pores of other Bodies, without partaking of their Motion, it is necessary that all Bodies should have all their Pores extended in right Lines parallel to the Direction of Motion; *viz.* that there may be no Reflections of the subtile Matter against the Sides of the Pores; otherwise the Matter, although ever so subtile, will be moved together with the Body, whose Pores it is supposed to fill. The subtile Matter therefore cannot but partake of the Motion of the Body, unless the Body moved be so disposed, that it has its Pores parallel to the Direction of Motion; but since its Situation may be varied infinite other ways, that is, the Lengths of the Pores may be inclined to the Line of Direction in infinite Angles; and therefore all these things being supposed, whilst the Body is moving, the subtile Matter placed in its Pores will be moved along with it: therefore a subtile Matter cannot so freely pervade the Pores of Bodies, but it must partake of their Motion; consequently the Body being moved, the Matter also contained in it will be moved, however subtile it be. If therefore a Body of Cork is moved, it carries along with it the Matter contained in its Pores; so that since it has a less Momentum than a Globe of Lead of the same Magnitude, and moved with the same Velocity, the Quantity of Matter in the Body of Cork will be less, and consequently there will be more Pores or Spaces absolutely empty.

FROM what has been demonstrated, may be deduced the following Theorem.

THEOR. IX.

THE Weights of all sensible Bodies near the Superficies of the Earth, are proportionable to the Quantities of Matter contained in them.

FOR, as is manifest from many Experiments of Pendulums, all Bodies falling perpendicularly by the Force of Gravity (abstracting the Resistance of the Air) describe equal Spaces in the same Time. For *in vacuo*, or a Medium that has no Resistance, the smallest Feather takes up no more time in falling than a heavy Piece of Lead; so that the Velocities of all Bodies falling in a given Time, are equal; their Momenta therefore will be proportionable to the Quantities of Matter contained in them: but the Forces generating Motion are always proportionable to the Motions or Momenta generated, and therefore in this case they will be as the Quantities of Matter in the Bodies moved; but the Forces that generate these Motions, are the Gravitations of Bodies, that is, their Weights. Therefore the Weights of all Bodies are proportionable to the Quantities of Matter contained in those Bodies. Q. E. D.

Cor. 1. THE Weight therefore of any Body, will be augmented or diminished, from the Quantity of its Matter being only augmented or diminished.

Cor. 2. WHEREFORE the Quantity of Matter continuing the same in any given Body, its Weight will also continue the same; and however its Figure, or the Texture of the Particles composing that Body be varied, yet its Weight will not be changed: so that the Weight of no Body depends on its Form or Texture.

SINCE [by Axiom 14.] the Nature of all Matter is the same, neither does one Body differ from another, but in its Modes, as by the Figure of its Parts,

Parts, Situation, and the like Forms; the Affections of Bodies, which do not depend on their Forms, will be the same in all Bodies: so that since (as has been said) the Weights of Bodies do not arise from their Forms, but depend on their Quantity of Matter; in equal Quantities of Matter, at the same distance from the Earth, the Gravitations towards the Earth will be equal: but if the Weights of two Bodies are unequal, the Quantities of Matter contained in them will be also unequal.

LET us suppose now two Globes of equal Magnitudes, the one of Lead, the other of Cork; if the Quantity of Matter in both was the same, (by what has been shewn) both Bodies would equally ponderate: for the subtlest Matter occupying the Pores of the Cork, would ponderate equally with the Matter of Lead that is equal to it. But since there is a great difference in the Weights of these two Bodies, there will be also a great difference in the Quantity of their Matter; and if Lead is thrice heavier than Cork, the Matter contained in the Lead will be triple of that in the Cork: so that there will be more Pores or Spaces absolutely empty in the Cork, than in the Lead. A *Vacuum* therefore is not only possible, but actually given; which was to be proved. And hence it follows, that the Quantity of Matter in any Body may be properly estimated by its Gravity.

SINCE the Momentum may be augmented, as well by increasing the Quantity of Matter, the Velocity remaining the same, as by increasing the Velocity, the Quantity of Matter remaining the same; the Antients (who were ignorant of the Force of Gunpowder to give a swift Motion to Bodies) to beat down their Enemies Walls, made use of Machines so framed, that a huge Mass of Matter might batter the Walls, though not with a great Velocity, yet with a violent Impetus: but now-a-days small Bullets are hurled

out of Cannon with a great Velocity, by the Explosion of Gunpowder. But although the warlike Engines of the Antients were much inferiour to ours, yet their Force to overturn Walls was almost incredible: for their Battering-Rams were composed of vast Beams joined together, whose Weight may be gathered from hence, that some of them required six thousand Men (*viz.* that some might succeed others) to direct them, and to give them their Motion. That part, whereby the Wall was battered, was made of solid-Iron, and they were so slung by Ropes, (I mean the compounded Rams) that their Lengths were parallel to the Horizon; whence being drawn backwards by a great number of Men, and immediately driven forwards by their Weight, and the Forces of the Men acting together, they struck the Walls with their Iron End: and, according to the Testimony of *Josephus*, there were no Towers so strong, or Walls so thick, as to sustain their constant Shocks.

IN Machines that raise Weights by the Circumgyrations of Wheels, sometimes the Wheels are rendered heavier by the addition of Lead, *viz.* that the greater Quantity of Matter may receive the greater Impetus or Quantity of Motion; whereby it may the better oppose the Resistance, arising as well from the Air as from the Friction of the Matter, and the longer preserve their Motion; which therefore once begun, will be easily continued.

FROM the same Principle also it is, that Spinners by putting on heavy Wharls on their Spindles, their Gyration is continued longer: since the Part of the Motion lost by the Resistance of the Air, bears a less proportion to the Motion increased by the addition of Matter, than that which it had to the Motion not thus increased.

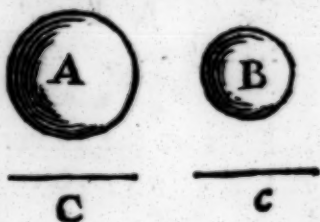
FROM what has been said may be solved the following Problem.

P R O B L.

PROBL. I.

To find the Velocity, wherewith a given Body must be moved, so that it may have a Momentum equal to any given Momentum.

LET the given Body be A, whose Momentum ought to be equal to the Momentum of the Body B, moved with the Celerity c ; let it be as A to B, so the Celerity c to another Quantity C; this will be the Velocity sought, wherewith if the Body A is moved, its Momentum will be equal to the Momentum of the Body B, as is manifest from the third Corollary of the third Theorem.



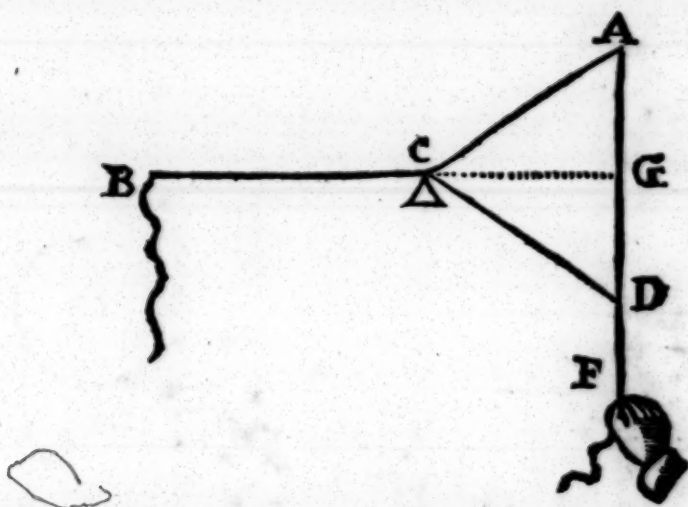
For the Momenta of Bodies are equal, if their Celerities are reciprocally proportionable to those Bodies; but, by Hypothesis, the Celerity of the Body B is to the Celerity of the Body A, as the Body A to the Body B: whence the Momentum of the Body A will be equal to the Momentum of the Body B.

Q. E. I.

AND hence it follows, that a Body, tho' ever so small, may have a Momentum equal to the Momentum of a Body, tho' ever so great, that is moved with a given Velocity. On this Principle depend all the Engines which are contrived to draw or raise Bodies: for if the Engines are so disposed, that the Velocity of the Power shall be to the Velocity of the Weight, as that Weight is to the Power; I say, in that case the Power will sustain the Weight. We will illustrate this in the five simple mechanick Machines. And first of all in the Lever, which we shall here consider as an inflexible Line, whether a right one or a curve, or composed of many right Lines, that is

cause, the way passed through by the Power B, according to its proper Direction, is to be estimated by βE . But by reason of the equi-angular Triangles αCF , βCE , αF is to βE as αC or AC to βC or BC ; that is, the Spaces run over by Powers according to their proper Directions, will be as their Distances from the Fulcrum.

BUT if the Direction of the Power is not a right Line, perpendicular to the Brachium of the Lever AC , let from the Fulcrum to the Line of Direction be drawn the Perpendicular CG , and the Space described by the Power according to its Propension, will be proportionable to that Perpendicular: for it matters not, whether the Thred FGA , by which



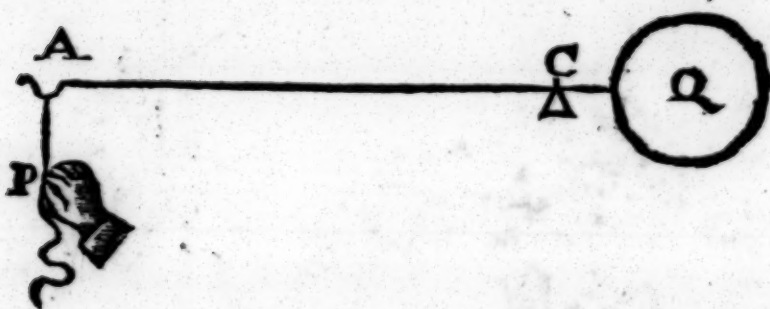
the Power acts, is affixed to the Point G or A, or indeed to the Point D; for the Line of Direction remaining the same, its Force to move round the Plane $ADCB$ will be the same, as if the Thred was fixed to the Point G, and the Way described by it in a given Time, according to its proper Direction, will be proportionable to the right Line CG . Wherefore it is manifest in every case, that the Way de-

described by any Force according to its proper Direction, is proportionable to the Distance of the Line of Direction from the Fulcrum.

THEOR. XI.

In a Lever, the moving Force or Power that has to the Weight the same Ratio, which the Distance of the Line of Direction of the Weight from the Fulcrum, has to the Distance of the Direction of the Power from the Fulcrum, will sustain the Weight; and therefore if it be ever so little increased, it will raise the Weight.

It is manifest from the preceding Theorem, that the Spaces which are described by a Power and Weight according or contrary to their proper Propensions, are proportionable to the Distances of the Line of Directions from the Fulcrum; but the Velocities are proportionable to these Spaces, and consequently will be also proportionable to the Distances. If therefore the Power P is to the Weight Q

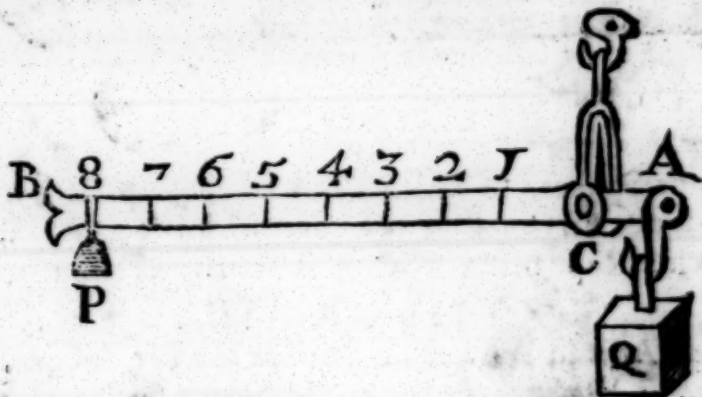


as CQ , the Distance of the Direction of the Weight from the Fulcrum to CA , the Distance of the Direction of the Power from the Fulcrum, the Power will be to the Weight, as the Velocity of the Weight to the Velocity of the Power; the Momentum therefore of the Power [by Cor. 3. Theor. 3.] will be equal to the Momentum of the Weight: and consequently, the

the Power will be equivalent to the Weight; which if it be ever so little increased, it will raise the Weight.

Q. E. D.

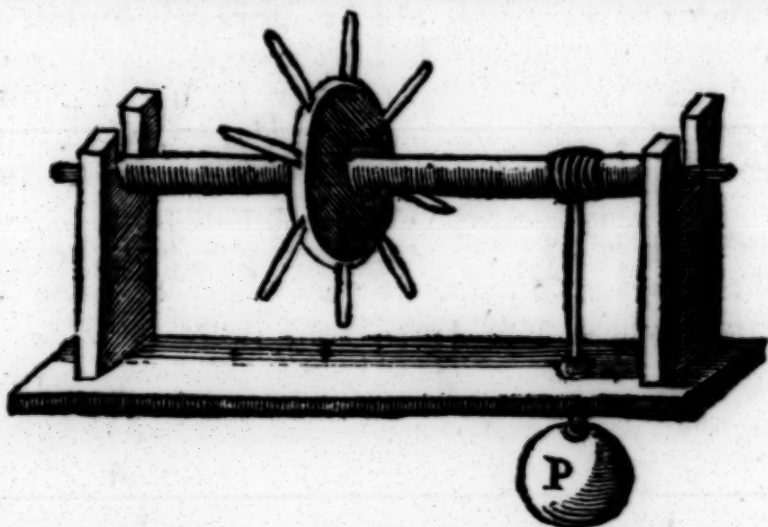
HENCE appears the Reason, why by the *Statera Romana* or Steel-yard, as it is commonly called, the Weights of different Bodies are examined all by one and the same Weight only. For this Instrument is a Lever of unequal Brachia, one whereof, CA, is extended in length, from the Axis of Motion C, (and which ought to be the Axis of Equilibrium) suppose one Inch, or less; the other Brachium, CB, may be of any greater length, that is capable of being exactly divided into Parts, each equal to CA, and



numbered by the Figures 1, 2, 3, 4, 5, &c. Let the Body whose Weight Q we want to discover, be hanged on at A; the given or known Weight P is moveable on the contrary Brachium, and by removing it from or bringing it nearer the Center C, is discovered the distance where is an exact Equilibrium: as, for example, if the Weight P at the Distance 8 equiponderates the Weight Q in A, it is thence concluded (by reason the Weights are reciprocally proportionable to their Distances) that the Weight Q is eight times the known Weight P.

Defin. THERE is a mechanical Instrument proper to raise Weights, called an *Axis in Peristrochio*. It is composed

composed of a Cylinder, that is named the *Axis*, supported at each end by *Fulcra*. About this Cylinder is fixed a Wheel, (which they call *Peritrochium*) and in its Circumference are fastened several Han-



dles or Spokes, to which a Force being applied, turns about the *Peritrochium* together with the *Axis*; about which a Rope being wound, raises the Weight.

THEOR. XII.

IN an Axis in Peritrochio (and in the like Instruments, whose Reason is the same) the moving Force, that has to the Weight to be sustained the same Ratio, as the Circumference of the Axis to which that Weight is applied, has to the Circumference of the outer Orb to which the Power is applied, will be equivalent to the Weight; which therefore being ever so little increased, will raise the Weight.

IT is manifest from the Make of the Instrument, that in one Conversion the affixed Weight P will be raised to an height that is equal in length to as much of the drawing Rope, as will once encompass the Axis; which therefore is supposed to be equal to its Circumference: and in the mean while the Power applied

applied to the Extremity of one of the Spokes, will have proceeded through the Circumference of the outer Orb, described by the Power in the same Revolution of the Instrument; (that is, the Space passed over by the Power in the same Time, will be equal to the Circumference of the outer Orb,) so that the Velocities of the Power and of the Weight, which are as the Spaces passed over in the same Time, will be as the Circumference of the outer Orb, and the Circumference of the Axis. Wherefore if the Weight is to the Power, as the Circumference of the outer Orb is to the Circumference of the Axis, the Velocity of the Power will be reciprocally to the Velocity of the Weight, as the Power to the Weight. Therefore [by Corol. 3. Theor. 3.] the Momentum of the Power will be equal to the Momentum of the Weight; and consequently, the Power will be equivalent to the Weight, and that Weight will be sustained by the *Axis in Peritrochio*: but if the Power be increased, or the Weight lessened ever so little, the Power will raise the Weight. *Q. E. D.*

Cor. BY how much the greater is the Compass of the outer Orb, that is, by how much the longer the Handles or Spokes are, or by how much the less the Axis is, by so much the more powerful will be the Force to raise the Weight.

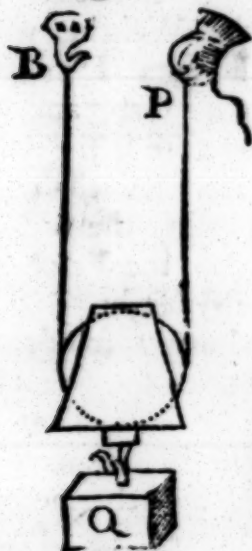
Defin. AN Instrument composed of one or more Wheels aptly disposed, moveable about their Axes, and about which Wheels a Rope being put, draws up the Weight, is called a Pulley.

THEOR. XIII.

IN a moveable Pulley, from the Position of the Wheels may be computed what the Force ought to be, which will be equivalent to an affixed Weight; namely, that Force, which is to the Weight, as 1 to the Number of Ropes, by which the Weight is suspended, will be able to sustain that Weight: which Force being therefore ever so little increased, will raise the same Weight.

Fig.

Fig. 1. LET one of the Ends of a Rope be fastened to a Hook B, and in its Duplication let be hung a moveable Pulley, to whose Block let be fixed the Weight Q; it is manifest, that in order to raise the

Fig. 1.*Fig. 2.**Fig. 3.*

Weight

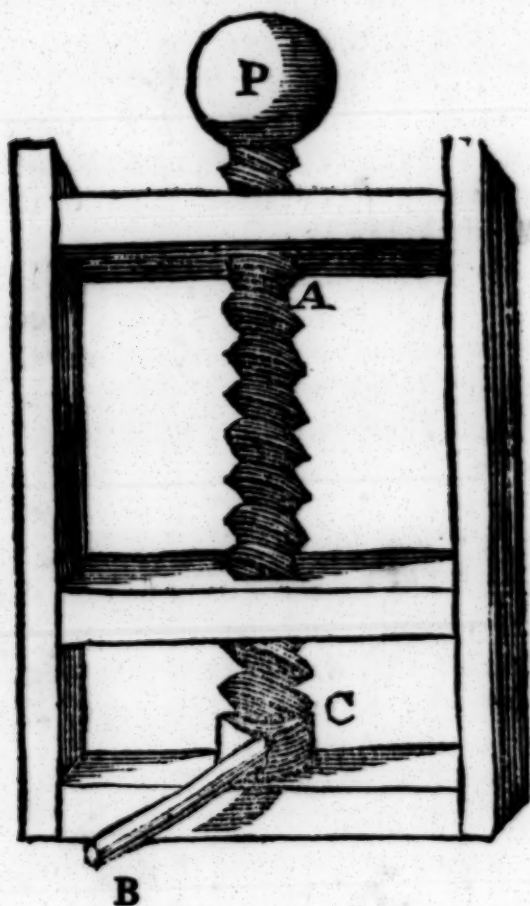
Weight *Q* one Foot, each Rope sustaining the Block together with the Weight, (reckoning downwards from the Hook) ought to be shorten'd also one Foot: that is, to raise the Weight one Foot, the Power must move through two Feet; whence in this Instrument, the Way of the Power will be double that of the Weight; and therefore the Celerity of the Power will be likewise double that of the Weight: so that if the Power is to the Weight as 1 to 2, its Momentum will be equivalent to the Momentum of the Weight, and it will sustain the Weight.

Fig. 2. If the Wheels are so disposed, that the Weight *Q* hangs from three Ropes; and that the Weight may ascend one Foot, all the three Ropes (we call them three, tho' indeed they are but the Continuation of one uninterrupted Rope) must be shortened one Foot each, which can be done no otherwise, than by the Power *P* being moved through the Space of three Feet. Whence since in this Instrument the Way of the Power is triple that of the Weight, its Celerity will be triple the Celerity of the Weight; so that if the Power is to the Weight as 1 to 3, its Momentum will be equivalent to the Momentum of the Weight.

Fig. 3. By the same way of reasoning, from the third Figure it is manifest, the Power *P*, which is but the fourth of the Weight *Q*, will be equivalent to it. In all Cases, the Power that is equivalent to the Weight, if it be ever so little increased, or the Weight diminished, the Power will then raise the Weight. *Q. E. D.*

Defn. A RIGHT Cylinder indented equally in a helical Form, is called a *Skrew*, whereof there are two sorts, *Male* and *Female*. It is named a male Skrew, when the convex Surface of the Cylinder is cut in; and a female Skrew, when the concave. The first is also called the outside Skrew, as the other is called the

the inside one. But the female Skrew ought to be so fitted to the male, that their Parts may answer to each other, (*viz.* that the Eminences of the one do agree with the Cavities of the other) whereby, if the



female Skrew is fixed, the male may be moved quite through it; or if the male Skrew is fixed, the female may be freely moved over it. Screws are chiefly employed to remove Obstacles, to break in pieces or to compress any thing, and to effect other Motions that are to be produced by Pulsion: the Power is applied to an Handle or Handspike.

THEOR.

THEOR. XIV.

IN the Skrew, if it be as the Compass, which the Force or Power applied, passes over in one Revolution of the Skrew, to the Distance between two helical Threds that are contiguous, (reckoned according to the Length of the Skrew) so is the Weight or Resistance to the Power; then the Power will be equivalent to the Weight, and that Power ever so little increased, will move the Impediment.

LET CA be a male Skrew, which being turned round by means of the Handle CB, may be passed through a female Skrew, and at the same time raise a Weight P. It is manifest by inspecting the Instrument, that in one Revolution of the Skrew, the Weight will be raised as much, as is the Distance between two contiguous Threds; and that the Power will pass over a Space equal to the Compass it describes in one Revolution: that is, the Way of the Weight will be to the Way of the Power gone through in the same time, as the Distance between two Threds to the Compass described by the Power in one Revolution, so that the Celerity of the Weight will be to the Celerity of the Power in the same Ratio: and therefore if it be, as the Power to the Weight, so the said Distance between two near Threds to the Way described by the Power, then the Power will be equivalent to the Weight; which being ever so little increased, it will overcome the Resistance. Q. E. D.

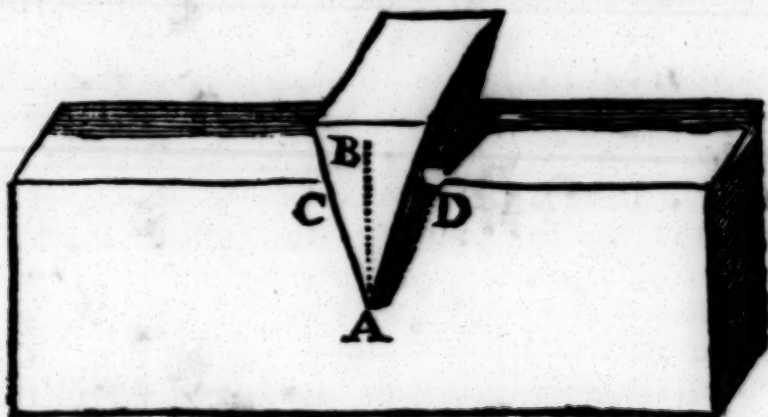
Defin. A WEDGE made of Iron, or some hard Substance, is frequently in use. Its form is that of a Prism of a small Height, whose opposite Bases are Isosceles Triangles; the Height of these Triangles is the Height of the Wedge, the Base of the Triangle is also called the Thickness of the Wedge, and the

right Line which joins the Vertices of the Triangles, is the Edge of the Wedge; and the Parallelogram that joins the Bases of the Triangles, is called the Back of the Wedge.

THEOR. XV.

A POWER applied directly to the Back of a Wedge, which is to the Resistance that is to be overcome by that Wedge, as the Thickness of half the Wedge to its Height, will be equivalent to the Resistance; and therefore if it be increased, it will overcome that Resistance.

LET the Resistance to be overcome by a Wedge be, for example, the Tenacity or Toughness of Wood, or any other Obstacle that is to be cleaved by a Wedge; it is manifest, that when the Wedge is driven in as far as it appears in the Figure, the



Way of the Power, or the Length run through according to its Propensity is BA; for so far, and no farther, is it gone. And after the same manner, the Way of each Impediment is half CD, and when the Wedge is drove through its whole Height, the Obstacle is divided by the whole Breadth of the Wedge; and consequently each part of the Obstacle is moved half the Breadth of the Wedge; and proportionably in its whole Progress, as appears from the Nature of

a Triangle. Whence, if the Power be to the Resistance on each side the Wedge, as half its Thickness to its Height, the Resistances will counterbalance the Power; so that if the Power be increased, it will overcome the Resistance.

SCHOLIUM.

Hence it appears, that the Force of the Power is not increased by the means of mechanical Instruments, which is impossible to be done; but by the application of the Instrument, the Velocity of the Weight that is either to be raised or drawn along, is diminished in such a manner, that the Momentum of the Weight becomes less than the Momentum of the Power. So, for example, if any acting Force is able to raise a given Weight of one Pound with a given Velocity, it is impossible, by the means of any Instrument whatever, that the same Force should be made to raise a Weight of two Pounds with the same Velocity; yet it may, by the help of an Instrument, be able to raise a Weight of two Pounds with half that Velocity; nay, the same Power will raise a Weight of a thousand, or even ten thousand Pounds, with the thousandth or ten thousandth part of that Velocity; but the Force of the Power is not thereby increased: the Motion only that it produces in raising that great Weight, is equal to the Motion that is produced when a Weight of one Pound is raised.

From what has been said, the Reason is plain, why in Canals of different Capacities, which communicate with one another, the Equilibrium of Liquors is preserved. For let A B C D be a large Canal communicating at C with a smaller one M N K H; if Water be poured in, it will rise in both Canals to the same height, and the Endeavour to descend, or the Force that the Water in the Canal F H has to flow through the Orifice C, is equal to the Force of the



Water in the Canal A C to descend through the same Orifice. For if we suppose, that the Water has descended in the Canal A C through the Height A I; it is necessary, that the Water in the Canal F H do rise to the Height H N, namely, to such a Height, that the Cylinder of Water M F G N be equal to the Cylinder A I L D, viz. the Cylinder of Water, which descends in the Canal A C: but in equal Cylinders, the Bases and Heights are reciprocal [by 15 *El.* 12.] that is, it will be

F M to A I, as the Orifice A D to the Orifice M N or F G; but it is F M to A I, as the Velocity of the Ascent of the Water in the Canal F N, to the Velocity of the Descent of the Water in the Canal A C; and the Orifice A D is to the Orifice M N, as the Water in A C to the Water in the Canal F H, for Cylinders of the same height are as their Bases. Whence the Velocity of the Water ascending in the Canal F H, will be to the Velocity of the Water descending in the Canal A C, as the Water in the Canal A C to the Water in F H; that is, the Velocities of the Waters are reciprocally proportionable to the Quantity of the Waters themselves, and therefore the Momenta of the Waters will be equal: but they are contrary, whence there will follow no Motion.

HENCE, by the bye, appears the Reason why Water, or indeed any Fluid, flowing out of a larger into a narrower Channel, is moved with a greater Celerity.

HENCE

HENCE in an Animal Body, if the small Branches of the Arteries, or the Capillary Arteries, have the Sum of their Orifices, or rather of their transverse Sections, greater than the transverse Section of the great Artery or Aorta, from which they all arise; the Velocity of the Blood will be less in the Extremities of the Body, than in the Aorta: but if this Sum be equal to the transverse Section of the Aorta, the Velocity of the Blood in the Extremities will be equal to the Velocity of the Blood in the Aorta; if the Sum is less, the Velocity of the Blood circulating through the extreme Arteries, will be greater than in the Aorta.



LECTURE XI.

Of the Laws of Nature.

WE have hitherto given the Demonstrations of Theorems relating to the Quantity of Motion, to Spaces passed over by moving Bodies, as likewise the Corollaries that might be deduced from them; now then we shall proceed to the Laws of Nature: namely, such Laws, as it is necessary that all natural Bodies do obey. And we shall deliver these in the same Order, and in the very same Words, as they are laid down by the Illustrious Sir *Isaac Newton*; the first whereof is this:

L A W I.

EVERY Body will continue in its State of Rest, or of moving uniformly in a right Line, except so far as it is compelled to change that State by Forces impressed.

SINCE natural Bodies consist of a Mass of Matter, that of itself is not able to induce any Change in its State; if Bodies were once at Rest, it is necessary that they should always remain in that State of Rest, unless there is applied a new Force to produce Motion in them: but if they were in Motion, the same Energy or Force would always preserve the Motion; and therefore Bodies would always retain their Motion, and would always proceed forward in the same right Line with the same Tenour, since they cannot of themselves acquire either Rest, or a Retardation, or a Change of their Direction to turn on one side or the other. There are some Philosophers, who readily enough acknowledge that no Body can move of itself, that is, pass from Rest to Motion of itself; but then they are not as willing to grant, that Bodies once moved cannot of themselves arrive at Rest, by reason they see the Motions of Projectiles languish by degrees, and at last the moving Bodies themselves come to Rest.

BUT as no Mode, or Accident, can of its own accord or by itself be destroyed, and as all Effects produced by transient Causes do remain always, unless there be some new and extraneous Cause that destroys them; so likewise Motion once commenced will be continued always, unless it is hindered by some external Cause: nor is it more in the power of a Body once moved, to lay aside its Motion or Energy to move, and return of itself to Rest, than it can put off the Figure that it has been once formed into, and acquire a new one, without some extrinsecal Cause.

THERE is besides in all Bodies a certain Force, or rather Inactivity, whereby they oppose every Change; whence it is, that they are very difficultly put out of their State, whatever it is: but that Force is the same in moving Bodies, as in those at Rest,

Rest, nor do Bodies less resist the Action, whereby they are brought from Motion to Rest, than that whereby they pass from Rest to Motion; that is, there is not required a less Force to put a stop to the Motion of any Body, than was before necessary to impress that Motion on the same Body. Whence, since the *Vis Inertia* or Inactivity of Matter always equally resists equal Changes, it will not be less powerful to continue a Body in Motion, which has begun to move, than to preserve a quiescent Body in the same State of Rest.

THERE are some Philosophers who suppose Body of its own nature to be as indifferent to Motion as to Rest; but by this Indifference they do not, I believe, mean such a Disposition in Bodies, whereby they do not in the least resist Rest or Motion: for on this Supposition it would follow, that any Body, however great, and moved with the swiftest Velocity, might be stopped by any the smallest Force; or if the great Body was at rest, it might be moved by any Body, however small, without the least Loss of Velocity in the impelling Body: that is, any small Body impinging on a greater one, would carry that greater Body along with it, without the least Loss of its Motion; and each Body after the Impulse would be jointly carried along with that Celerity, that the small Body had at first, which, we all know, is absurd. This Indifference therefore is not placed in a Non-resistance to Motion from a State of Rest, or to Rest from a State of Motion; but in this only, that a Body of its own nature is not more propense to Motion than to Rest, nor more resists to pass from a State of Rest to Motion, than to return again from that Motion to the same State of Rest: besides, any quiescent Body may be moved by any Force, and an equal Force acting in a contrary Direction, will be

able to destroy that Motion; and in this, they would have this Indifference placed.

SINCE, according to this Law of Nature, a Body once in Motion always continues in that Motion, the Philosophers ask why all Projectiles lose by degrees their Motion? (which Motion they are pleased to call violent.) Why do they not proceed *in infinitum*? If Motion did not of its own nature decay, a Stone thrown at the beginning of the World would by this time have gone through an immense and almost infinite Space. And so indeed it would, if its Motion had been *in vacuo* or in free Spaces, and without any Gravity. But since all Projectiles are carried either through the Air, or on the rough Surfaces of other Bodies, they must be necessarily retarded: for since all Bodies in motion must drive and thrust out of its place the resisting Air, or overcome the Roughness of the Superficies upon which they are moved; they will lose all that Force and Motion, that is constantly employed in overcoming these Obstacles, and consequently the Motion of Projectiles will be continually diminished: but if there was no Resistance in the Medium, no Roughness in the Superficies on which they were moved, no Gravity that continually forces the Bodies towards the Earth, Motion would always continue the same, without any Retardation at all. So in the Heavens, where the Medium is exceedingly rare, the Planets do continue in their Motions for a very long time; and upon Ice, or any other very smooth Surface without any Roughness, heavy Bodies in motion are not soon brought to rest.

LET the Philosophers at length leave off inquiring into the Cause of the Continuation of Motion; for there is no other besides that first Cause, which does not only preserve Motion, but every thing else in its *Being*, namely, the All-wise and Great God. Nor is Motion continued any other way, than that where-
by

by is preserved the Figure, the Colour, and any other the like Affections of Body, which always remain the same, unless they are altered by any external Force.

THESE Philosophers would have done much better, and more suitable to the Rules of good Method, if they had inquired out the Reasons of the Decay or Loss of Motion: but there are some so blind in this Affair, that they suppose that very thing to be the Cause of the Continuation of Motion, from which indeed proceeds its Decay.

LET the Philosophers also leave off disputing concerning the Communication of Motion; for from what has been said, may be understood why a Stone goes with so great an Impetus out of the Hand of one that throws it; namely, when the Stone is held in the Hand, it is necessary that it partakes of the Motion of that Hand [by Axiom 8.] so that it will be carried in the same Direction with the Hand: but every natural Body once moved, continues in the same Motion [by the above-mentioned Law] until it is hindered by some external Agent, whence when the Person that casts the Stone draws back his Hand, but not drawing back the Stone, it will proceed right forwards. After the same manner, if a Ship or Boat is swiftly driven by the Winds or Oars, those that sit in her, have the same swift Motion communicated to them; but if the Ship stop of a sudden, every thing that is placed in that Ship endeavours to continue its Motion, and such things as are not firmly fixed to the Ship, do after the Ship is at rest, leaving their places, still move on; and hence there is a danger, lest Men relatively at rest in a Ship, after so sudden and as it were violent Change of its State, are not thrown along; namely, since the Motion which they before received from the Ship, is not yet destroyed.

IF a Stone is swiftly whirled about in a Sling, it describes a Circle with the same Velocity as the bottom of the Sling has, wherein the Stone is placed; but since every Body affects to proceed in a right Line, the Stone in every Point of its Orb would go out in a right Line, touching that Orb in the Point wherein it is placed, if it was not hindered by the String: so that if the String be let go, broke, or any other ways desists from detaining the Stone, the Stone would no longer move in a Circle, but in a right Line, excluding the Motion arising from its Gravity.

THAT Endeavour, which the Stone thus whirled about has in any Point of its Orb, of running out in the Tangent, stretches a String by which it is detained in that Orb; and that Force, whereby the String is stretched, arises from the centrifugal Force, whereby, namely, it endeavours to recede from the Periphery. This Tension any one may easily experience in a Sling; and it is found by Observation, that the swifter the Stone is whirled about, or the greater the Weight is that is put into the Sling, the greater will be the Tension of the String, or it will be stretched the more,

FOR this reason, some Philosophers will have this centrifugal Force to proceed from Gravity alone, but neither Reason nor Experience favours this Opinion; for in a Sling, the String is not only stretched when the Stone moves through the lowest part of its Orb, but likewise when it describes the upper part: which cannot arise from Gravity, since Gravity in the upper part of its Orb can only draw the Stone towards the Center, which is directly contrary to the centrifugal Force, that compels it to recede from the Center. Besides, if the Stone is whirled about in a Circle parallel to the Plane of the Horizon, the String is then also stretched; but Gravity can no
ways

ways produce that Tension, since the Stone is neither carried upwards or downwards, whose Motion can therefore be neither increased or diminished by this Gravity: the centrifugal Force consequently does not arise from Gravity, but only from that Endeavour, which all Bodies have of proceeding in the same right Line.

If we suppose the Earth to revolve about its Axis, all of us who live upon its Surface must revolve together with it; so that if its Motion should suddenly cease, all things not firmly fixed to it, would be thrown from it with a violent Motion. So likewise if it is carried with an annual Motion about the Sun, and that Revolution should be suddenly stopped, all things would be thrown from it, and, like Planets, would revolve about the Sun, by the same Cause, as the Earth itself was moved before about the Sun.

SINCE the Earth turns about its Axis, and all things on it describe Circles parallel to the Plane of the Equator, the Philosophers demand how it comes to pass, that all Bodies are not thrown off from its Surface, when by the Law of Nature all Bodies affect a Motion in a right Line? So indeed they would be thrown off, if they were not detained on the Earth by another Force, which is Gravity, and which is much more potent than the centrifugal Force.

If a Vessel full of Water is placed on any horizontal Plane, and suddenly pushed forward by any great Force, the Water in the Vessel will at the beginning be seen to tend towards the Parts that are contrary to the Motion of the Vessel: not that in reality such a Motion is impressed on the Water, but since it endeavours to continue in the same State of Rest, the Vessel cannot immediately communicate its Motion to the Water contained in it; and therefore the Water deserted by the Vessel, and truly at rest, will
seem

seem to change its relative place. But at length after the Motion of the Vessel is communicated to the Water, and that together with the Vessel begins to move uniformly and with the same Celerity, if then the Vessel be suddenly stopped, yet the Water will endeavour to continue in the same Motion, and rising up the Sides of the Vessel, part of it will even run over.

IF a Ship is tossed by a Storm and a troubled Sea, the Men that are sitting in her, and relatively at rest, will be seized with Pains, Sicknes, a Nausea and Vomiting, especially if they have not been used to the Sea; since the Liquors contained in their Stomachs, Intestines, Blood-Vessels and other Ducts, do not immediately observe the Tossings of the Ship, whence the Motion of the Fluids in the human Body is disturbed, and Diseases do arise.

L A W II.

THE Change of Motion is always proportionable to the moving Force impressed, and is always made according to the right Line, in which that Force is impressed.

THIS follows from the fourth Axiom; for if any Force generates a certain Motion, a double Force will generate a double Motion, and a triple one a triple Motion: and this Motion, because it is determined always into the same Direction with the moving Force, (for it solely arises from it) it will be always made according to the same Direction [by the first Law,] nor can the Body decline according to any other Direction, unless some new Force opposes the former; so that if the Body was moving before, the Motion produced from the impressed Force will either be added to the conspiring Motion, or subtracted from the opposing Motion, or obliquely joined to the oblique Motion, and will be compounded with it according to the Determination of both.

If

IF any Force produces a Motion in a given Body, [by the first Law] that Body will always continue in its Motion: but if afterwards the same or an equal Force again acts on that same Body according to the same Direction, the Motion thence arising will be equal to the former, and therefore the Sum of the Motions will be double that former; if the same Force a third time acts in like manner on the same Body, the Motion hence occasioned will be also equal to the first, and therefore the Sum of the Motions will be triple the Motion first impressed; and if the same Force should act again on the same Body as before, the Sum of all the Motions will be quadruple the Motion first impressed, and so on continually.

HENCE if this new Force should act continually and equally, in equal Intervals of Times, the Motion thence arising would be as the Sum of the Times in which it was generated: so that, since by reason the Body is given, the Motion is as the Velocity, the Velocities so generated will be as the Times from the beginning of the Motion, and the Motion will be equally accelerated. Hence the following Theorems may be easily demonstrated.

THEOR. XVI.

IF Bodies at all Distances from the Earth gravitate equally, the Motion of Bodies falling by their Gravity in the same right Line, will be an equably accelerated Motion.

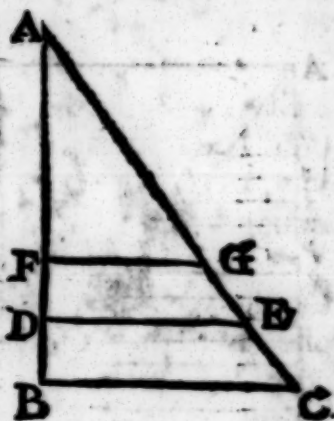
LET the Time in which the Body is falling, be supposed to be divided into equal and very small Particles, and let the Gravity acting in the first Particle of Time draw the Body towards the Center; if now after that first Time the Action of Gravity should cease, and the Body be no longer heavy, nevertheless the Motion received from the first Impulse would be always continued, and the Body would
equally

equally descend to the Earth, [by the first Law;] but since the Body is constantly heavy, and Gravity acts without intermission, also in the second Particle of Time the same Gravitation will communicate to it another Impulse equal to the former, and the Velocity of the Body after these two Impulses will be double the former Velocity; and if the Force of Gravity was quite removed, yet the Body would continue to move in the same right Line with the same Velocity: but since in the third Particle of Time the Body is acted upon by the same Gravity, it will also after that third Time acquire another Motion equal to each of the former; so likewise in the fourth Particle of Time, the Gravitation will add to the heavy Body a fourth Impetus equal to each of the former: and so of the rest. The Impetus therefore or Motions of a given Body acquired by its Gravity, are as the Particles of the Time elapsed from the beginning: so that since the Action of Gravitation is continual, if those Particles are taken infinitely small, the Motion of the falling Body acquired by its Gravity, will be as the Time elapsed from the beginning of the Fall; and since the Body is given, its Motion will be as its Velocity, therefore the Velocity will be always as the Time in which it was acquired. In-
somuch that in equal Intervals of Times, a falling Body will receive an equal Increase in its Velocity, and consequently its Motion will be uniformly accelerated. *Q. E. D.*

IN like manner, from the same Principles may be demonstrated, that the Motion of Bodies thrown upwards is equably retarded; since the Force of Gravity, acting constantly and equally against the Motion commenced, in equal Times will equally diminish its Motion, even till all the Motion upwards is quite destroyed.

Cor.

Cor. LET the right Line AB represent the Time in which a Body is falling, and BC making with AB a right Angle, represent the Velocity acquired at the end of the Fall; join AC, and through any Point D let be drawn DE parallel to BC: this will be as the Velocity acquired at the end of the Time AD. For (by reason of the similar Triangles ABC, ADE) AB is to AD as BC to DE; but BC represents the Velocity acquired in the Time AB, whence (since the Velocities are as the Times) DE will represent the Velocity acquired at the end of the Time AD: in like manner, FG will represent the Velocity in the Point of Time F, and in every Point of Time the Velocities will be as the right Lines drawn within the Triangle through it, and parallel to the Base BC.

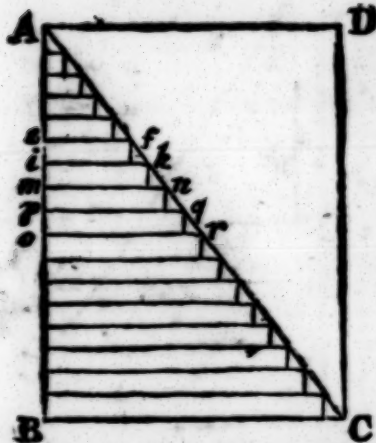


THEOR. XVII.

If a heavy Body descends from Rest with a Motion uniformly accelerated, the Space, that from the beginning of its Motion is run over by the Body in a given Time, will be half the Space that may be uniformly run over in that Time, with a Velocity equal to that which the heavy falling Body will have acquired at the end of that Time.

LET AB be the Time in which the heavy Body is a falling, and let BC be the Velocity acquired at the end of the Fall; compleat the Triangle ABC, and Rectangle ABCD; moreover, let the Time AB be distinguished into innumerable Particles, *ei, im, mp, &c.* draw *ef, ik, mn, pq, &c.* parallel to the Base;

Base; [by the preceding Corol.] ef will be as the Velocity of the heavy Body in the infinitely small



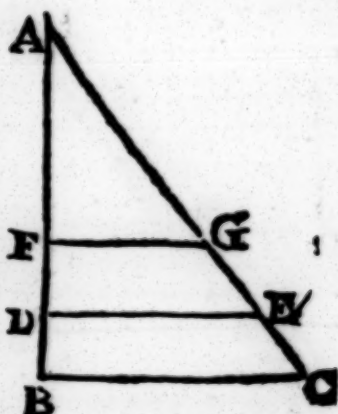
Particle of Time ei ; and ik will be its Velocity in the Particle of Time im ; also mn will be its Velocity at the Point of Time mp ; and so pq will be the Velocity in the Particle of Time po : but [by Cor. Theor. 7.] the Space passed over in any Time, and with any Velocity, is a Rectangle under that Time and Velocity: whence the Space

passed over in the Time ei with the Velocity ef , will be as the Rectangle if ; so the Space passed over in the Time im with the Velocity ik , will be as the Rectangle mk ; so likewise the Space passed over in the Time mp with the Celerity mn , will be as the Rectangle pn : and so of the rest. Wherefore the Space passed over in all these Times, will be as all these Rectangles, or as the Sum of all these Rectangles; but since the Particles of Time are infinitely small, the Sum of all the Rectangles will be equal to the Triangle ABC : but [by the Corol. of Theor. 7.] the Space passed over by a moving Body in the Time AB with an uniform Celerity BC , is as the Rectangle $ABCD$. Whence the Space passed over in a given Time by a heavy Body falling from Rest, will be to the Space passed over in the same Time with an uniform Velocity, equal to that which is acquired by the heavy Body at the end of its Fall, as the Triangle ABC to the Rectangle $ABCD$; but the Triangle ABC is half the Rectangle $ABCD$, whence the Space that is passed over in a given Time by a falling Body from the beginning of its Fall, will be

be half that Space which it would have passed over in the same time, with an uniform Velocity equal to that which it has acquired at the end of its fall. *Q. E. D.*

Cor. 1. THE Space which is passed over with the Velocity CB in a Time equal to half AB, will be equal to the Space passed over by a falling heavy Body in the Time AB.

Cor. 2. FROM the Demonstration itself it follows, that as the Space passed over in the Time AB may be represented by the Triangle ABC, so the Space described by the heavy Body in the Time AF may be represented by the Triangle AFG: also the Space gone through in the Time AD, may be expounded by the Triangle ADE.



Cor. 3. THE Spaces passed over, reckoning from the beginning of the Fall, are in a duplicate Ratio of the Times: for the Space passed over in the Time AB, is to the Space passed over in the Time AF, as the Triangle ABC to the Triangle AFG; but (by reason of the similar Triangles ABC, AFG) the Triangle ABC is to the Triangle AFG in a duplicate Ratio of the Side AB to the Side AF: so that the Space passed over in the Time AB will be to the Space passed over in the Time AF in a duplicate Ratio of the Time AB to the Time AF. The Spaces therefore passed over by a heavy Body from the beginning of its Fall, are as the Squares of the Times in which they are run through.

Cor. 4. HENCE if a heavy Body falling from a State of Rest, passes over a certain Space in a given Time, the Space passed over in a double Time will be quadruple the former Space; and the Space run

L

over

over in a triple Time will be nine times greater than the Space first run over, &c. that is, if the Times are taken as 1, 2, 3, 4, 5, &c. the Spaces described in these Times reckoning from the beginning of Motion, will be as 1, 4, 9, 16, 25, &c.

Cor. 5. SINCE the Space passed over in the first Time is as 1, in the second as 4, computing from the beginning, the Space described separately in the second Time will be as 3. After the same manner, since the Space described at the end of the third Time is as 9, and at the end of the second as 4, the Space described separately in the third Time will be as 5; and so of the rest: taking therefore equal Parts of the Times, the Spaces described separately in every one of those Times by a heavy Body falling from a State of Rest, will be as 1, 3, 5, 7, 9, 11, &c. namely, as the odd Numbers.

Cor. 6. HENCE also, since the Velocities acquired by a falling Body are as the Times, the Spaces passed over will be likewise as the Squares of the Velocities: and the Velocities, as well as the Times, will be in a subduplicate Ratio of the Spaces through which the heavy Body falls, computing from the beginning of the Motion.



LECTURE XII.

LAW III.

REACTION is always equal, and contrary to Action; or the Actions of two Bodies upon each other are always equal, and in contrary Directions. That is, by Action and Reaction equal Changes of Motion are produced in Bodies acting upon each other, and these Changes are impressed towards contrary Parts.

THIS Law will be better illustrated by Examples.

1. If one Body impinges on another Body at rest, whatever Motion is impressed on the Body at rest, just the same Quantity of Motion is

subtracted from the impinging Body. For example, if the Body A,



whose Motion is twelve Parts, is carried towards the Body B, and if after the Impulse, it shall have communicated to B five Parts of its Motion, then will there remain to A but seven Parts of Motion, so that the Changes which happen to both Bodies will be equal; and the Effect will be exactly the same, as if a Force equivalent to five Parts of Motion should push the Body B towards C, and another Force equal to the former should act on the Body A, and should press it the contrary way towards D.

2. If the Body B should not be at rest, but tend towards C, and the Body A moving faster, should impinge on it; the Body A would lose just the same

Quantity of Motion as the Body B would gain, and the Changes of Motion produc'd by the Impulse on both the Bodies, (that is, the Increase of Motion in one, and the Decrease in the other) will be equal.

3. IF the Bodies A and B meet each other, A being carried towards C with twelve Parts of Motion, but B towards D with three Parts of Motion; whatever Change of Motion happens to the Body B, the very same will befall the Body A: for example, if after the Concourse B is carried towards C with two Parts of Motion, the Change of Motion that is made in it will be five Parts; namely, equal to the Sum of two Motions, *viz.* of that whereby it was before carried towards D, and which was destroyed by the Impulse of the Body A, and of that which it then received, and wherewith it tends towards C; and the Motion lost in the Body A, will be exactly equal to these five Parts of Motion: so that (as in the first Example) the Effect will follow exactly the same, as it would be, if a Force with five Parts of Motion should push B towards C, and another Force equal to the former should be impressed on the Body A, and press it towards the Parts D.

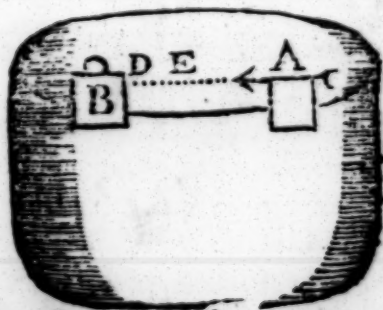
AND universally, the Magnitude of a Stroke which arises from the Concourse of two Bodies, is always equally receiv'd by both the Bodies; whence the Changes of Motion produced by the Stroke in both the Bodies, will be always equal.

So if an Iron Hammer strikes against a Glass, the Blow is received equally by the Hammer as by the Glass, and the Glass is broke, when at the same time the Hammer remains whole, not because the Force of Percussion impressed on the Glass is greater than that which is received by the Hammer, but because the Parts of the Iron being harder, and cohering more firmly together, do more strongly resist the same Force of Percussion, than the brittle and less cohering

cohering Parts of the Glass can do. Just after the same manner, if a Body is fasten'd to a Wall by a fine Thred, a small force will be sufficient to pull it from thence; but if the same Body is fasten'd by a strong Rope, the former Force equally applied, will little avail to draw it away.

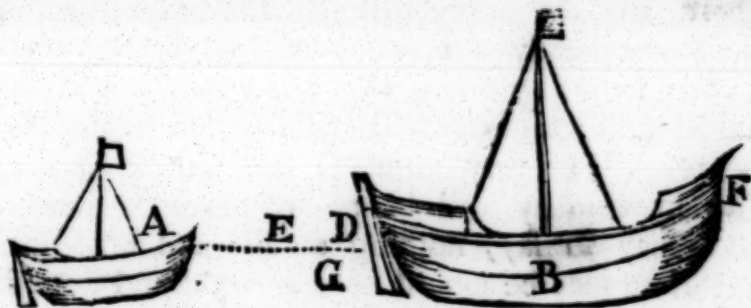
4. If a Horse draws forwards a Stone tied to a Rope, the Horse also will be equally drawn backwards towards the same Stone; for the Rope stretched both ways, by the same Endeavour of relaxing itself, will equally strain the Stone towards the Horse, and the Horse towards the Stone: whence the Forces of Attraction, as well in the Horse as in the Stone, will be equal; but since the Firmness and Force of the Horse standing upon the Ground is so great, that he is able to resist the drawing of the Rope, he will not in the least yield to the drawing Rope, nor by its Force be removed out of his place; but the Stone, wherein there is not so great a resisting Force, will be moved towards the Horse.

5. We find by Experience, that in magnetick Attractions the Loadstone does not only draw the Iron, but is in like manner equally drawn by the Iron: For let a Loadstone be placed on a piece of Cork B, and likewise the Iron A on another piece of Cork, and let them both swim in Water; then let the Loadstone be held by any one's Hand, and we shall see the Iron advance to the Loadstone: and if the Iron is kept immovable, we shall find the Loadstone will come to it: but if both the Bodies are permitted to float freely on the Water, we shall behold both the Loadstone and the Iron mutually



meet each other, and the Force of Attraction acts equally upon them both, producing equal Motions in each: I say, the Motions will be equal, but not the Celerities, unless the Iron and the Loadstone weighed alike; for if they are of different Weights, that which weighs the most, will have the least Celerity. For example, if the Loadstone is ten times heavier than the Iron, the Iron reciprocally will have ten times a greater Velocity; namely, that there may be generated equal Quantities of Motion in each Body: so that the Loadstone and the Iron will not meet in the middle Point E, but in the Point D, which will so divide the Distance BA, as BD will be to DA as the Weight A to the Weight B. So in the present Example, if BD is the eleventh part of the whole Distance, the Point D will be where the Loadstone and the Iron will meet each other: for since BD is the eleventh part of the Distance BA, BD will be to DA as 1 to 10; but as 1 to 10, so (by what was said before) will be the Velocity of the Body B to the Velocity of the Body A. Wherefore since the Spaces passed over in a given Time are proportionable to the Velocities, during the time the Body A is running over the Space AD, the Body B carried with the tenth part of the other's Velocity will have passed over a Space equal to the tenth part of the Space run over by the Body A: so that at the end of that Time it will be found in D, in which Point therefore the Loadstone and Iron will meet each other. After the same manner, two Loadstones being placed on two Pieces of Cork, if their corresponding Poles be set towards each other, they will mutually attract each other equally; but if their adverse Poles be placed near one another, these Poles will mutually fly from each other, and the Quantities of Motion produced by the Force wherewith they avoid each other, will be equal in both.

6. THE same thing may be shewn in other kinds of Attractions. For let A and B be two Barges upon the Water, and let a Man placed in one of them, as, for example, in A, draw by the means of a Rope the other Barge B towards



himself; by this Action of the Man, not only the Barge B will approach towards A, but A also will be equally drawn towards B; and the Quantities of the Motions produced by the Attraction, will be equal in each Barge: whence if the Barges are of the same Weight, they will have, *ceteris paribus*, equal Velocities, and they will meet at the middle Point E. But if one is greater than the other, that is, has in itself a greater Quantity of Matter, or a greater Weight, then that which is the biggest will have the least Velocity. For example, if the Barge B is ten times greater than the Barge A, the Velocity of A will exceed the Velocity of the Barge B ten times, and the Barges will meet in the Point G, which so divides their first Distance AD, that AG is ten times greater than GD; that is, GD will be the eleventh part of the whole Distance AD: but if B should be a Ship a thousand or ten thousand times bigger than A, its Velocity would be a thousand or ten thousand times less than the Velocity of A, and consequently it would be scarcely discernible. Now if B should be another Body infinitely great, its Ve-

locity would be infinitely small, that is, just nothing in respect of the Velocity of A. Hence if the Rope should be fastened to the Shore, and the Man in the Barge should draw by the Rope the Shore towards himself, the Barge would approach towards the Shore, and the Shore towards the Barge: but since the Shore firmly adheres to the whole Body of the Earth; its Magnitude, which is therefore equal to the Magnitude of the whole Earth, in respect to the Barge, will be very immense and almost infinite, so that its Velocity will be nearly infinitely small, and (as I may so say) nothing at all: and consequently the Shore may be look'd upon as fix'd and impossible to be mov'd, and the whole Velocity as altogether appertaining to the Barge. If the Weight of the Barge B is a thousand Talents, and is carried towards F with an hundred Degrees of Motion, the Momentum of that Barge will be [by Theor. 3.] an hundred thousand parts. Now if to the Barge B is fastened the Boat A, whose Weight is ten Talents, whatever Motion by this means is communicated to the Boat A, the very same Quantity of Motion will be taken from the Barge B. So that,

7. If any one in the Boat A draws the Rope A E, by which the Boat is fastened to the Barge B, in such a manner that the Boat moves forward with five hundred parts of Velocity, the Motion thence arising will be five thousand parts; and so much of its Motion the Barge B will lose, and consequently there will remain to it ninety-five thousand Parts of Motion, whence the Velocity of the Barge B will be ninety-five Parts.

8. If any one sitting in the Boat A, should by a Spreat, or the like Instrument, push or thrust the Barge B towards the Parts F, by that pushing the Boat A would recede towards the contrary Parts; so that the Quantity of Motion in both the Vessels,
arising

arising from the Force of the Man's pushing, would be equal: whence if the Barge B is ten times greater than the Boat A, it will have ten times a less Velocity; if it is an hundred times bigger, it will likewise have the hundredth part of the Velocity of the Boat A: so that if the Body B is an immense Body, the Velocity of the Boat A will be immense in respect of that which ought to be found in B. Whence if any one sitting in a Ship, should by means of a Pole, endeavour to thrust the Earth or the Shore from him, the Ship by his thrusting would recede from the Shore; for the Shore, in respect of the Ship may be looked upon as an immense and fixed Body, whose Velocity therefore will be exceeding small, or indeed none at all, in respect to that which is found in the Ship.

If the Boat E D G is rowed by Oars, since the Water is by the broad Ends of the Oars A, B, driven backwards towards the Parts C, that will again equally re-act upon the Oars, and force them, together with the Boat to



which they are fixed, towards the Parts H: for which reason alone it is, that the Boat is moved forwards; for if there was no Re-action, and the Water impressed no Motion on the Oars towards the Parts H, when it was thrust by the Oars the contrary way, the Boat would stand still, since there would be nothing that could force it towards the Parts H. But since the Water by its Re-action impresses as great a Motion on the Boat E D, as it receives itself from the Oars, it hence follows, the greater

greater the broad Ends of the Oars are, or the more they are in number, *cæteris paribus*, or else the swifter they are moved within the Water, the Boat will be carried with a greater Impetus.

HENCE since swimming is nothing else than a rowing with the Hands and Feet, it may be easily understood, why by swimming within the Water we are moved forwards; namely, since the Water is impelled backwards by the Palms of our Hands and the Soles of our Feet, that by its re-acting forces those that swim towards the contrary parts: so that the Motion generated in the Water, is equal to the Motion wherewith the Swimmers proceed forwards. It is just the same thing in the flying of Birds; for as the Birds by their Wings bear the Air downwards, the Air by its Re-action raises them upwards; and if they drive the Air towards the East, for example, the Re-action of the Air obliges them to tend towards the West. So Gunpowder within a Cannon being fired rarefies, and by its Force acts equally upon the Bullet and the Cannon out of which that Bullet is to be hurled; for the rarefied Air endeavouring to expand itself every way, will equally press the Cannon backwards, as the Bullet forwards, whence its elastick Force will produce equal Quantities of Motion in both; and by dividing these Quantities of Motion, as well by the Weight of the Engine as by the Weight of the Bullet, the Velocities thence arising will be reciprocally proportionable to the Weights.

SINCE all Bodies near the Superficies of the Earth gravitate towards the Earth, in like manner the Earth will gravitate towards each Body, and is attracted towards them; and the Motions generated by this Attraction will be equal, as well in the Earth, as in the heavy Bodies falling to the Earth: so if a Stone by the Force of its Gravity falls downwards
to

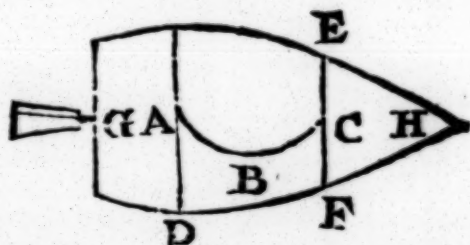
to the Earth, the Earth in like manner will rise towards the Stone; but since the Quantity of Matter in the Earth exceeds immensely the Quantity of Matter in the Stone, the Velocity of the Stone will exceed immensely the Velocity wherewith the Earth tends towards the Stone, so that (to speak physically) the Velocity of the Earth will be nothing: which will thus appear upon a Calculation. Let us suppose a Stone of an hundred solid Feet in magnitude to fall to the Earth: the Space run over by the Stone in a Minute's time will be about fifteen Feet; but (according to their Accounts who have written concerning the Bigness of the Earth) the whole terraqueous Globe contains 30 000 000 000 000 000 000 000 solid Feet. Let us now suppose the Earth to be every where of the same density with common Stones, (tho' it is probable it is much more dense) whence the Quantity of Matter in the Earth will be to the Quantity of Matter in the Stone of a hundred feet, as 300 000 000 000 000 000 000 000 to 1; and therefore whilst the Stone by its Weight falls through the Space of fifteen Feet, the Earth will be drawn towards the Stone the $\frac{15}{300\,000\,000\,000\,000\,000\,000}$ Parts of a

Foot: which is so very small a Quantity, as not to be conceived by the Imagination, and consequently may be neglected and esteemed as nothing in Philosophy; altho' speaking geometrically, and according to Truth, we ought to say that the Earth advanced towards the Stone, and that both the Bodies mutually attracted each other.

IF the Moon by its Gravity is retained in its Orb, so that it does not leave the Earth; that is, if the Moon gravitates towards the Earth, in like manner the Earth and all its Parts will gravitate towards the Moon: and hence arises the continual Flux and Reflux of the Sea. But this by the bye, for we shall in

in another place explain more fully the Motions of the Sea.

LET such a Ship be at rest in the Water, as may be easily moved by any external Impulse, yet no Force acting within that Ship, and solely contained in it, can communicate any Motion to it at all; for let G H be the Ship, and let any Machine, for ex-



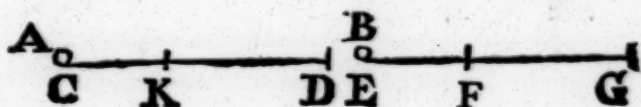
ample, the elastick or springing Body A B C, which being violently strained, will endeavour to unbend itself, be placed within the Ship: moreover, the Machine being compressed, the Side B C will approach the Side A B; the Spring by its natural Energy, or restoring Force, endeavouring equally to unbend itself on both sides, will equally impel the Obstacle D A towards G, and the Obstacle E F towards H: and therefore the Ship being impelled by these contrary and equal Motions, will not be moved at all. Just in the same manner, as if one standing at the head of the Ship H, should by a Rope draw to himself the Stern G; now the Rope being distended both ways, by its Endeavour to relax it self, it will equally urge the Stern towards the Man that draws, and the Man towards the Stern; and since he is seated at the head of the Ship, in like manner the Head will be equally drawn towards the Stern: whence the two contrary and equal Motions will destroy each other, and consequently no Motion at all will follow.

FROM this Law may be demonstrated the following Theorems.

THEOR. XVIII.

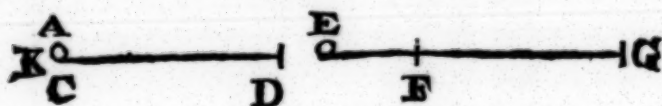
If one Body strikes against another that is either at rest, or moving in the same Direction, but slower; the Sum of the Motions in both the Bodies towards the same Parts, will remain the same after the Stroke, as it was before that Stroke.

LET the Body A move in the Direction CD from C towards D, and let it strike against another Body B, that is either at rest, or moves slower in the same

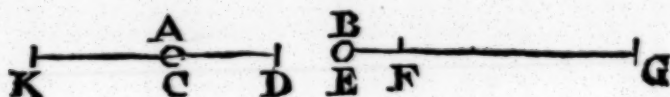


Direction: I say, the Sum of the Motions in both Bodies towards the same Parts, namely, from C towards D, before and after the Stroke, will remain the same. Let CD express the Motion of the Body A, and if the Body B moves, let the right Line EF represent its Motion towards the same Parts, and therefore the Sum of the Motions may be expressed by the Sum of the right Lines CD, EF. Now since Action and Re-action are always equal and contrary, the equal Forces impressed towards contrary Parts, will produce in both the Bodies equal Changes of Motion towards the contrary Parts: if therefore the Motion impressed on B by the Stroke of the Body A shall be represented by FG, the contrary and equal Force acting on the Body A will take as much from its Motion made towards the same Parts; so that by putting DK equal to FG, CK will be as the Motion of the Body A, and EG as the Motion of the Body B after the Stroke or Concourse; and therefore the Sum of the Motions will

will be as the Sum of the right Lines CK, EG. But since FG is equal to KD, if to both there be added EF and CK, then EG and CK will be equal to CD, EF: whence the Sum of the Motions towards the same Parts, will remain the same, both before and after the Stroke. If FG is equal to CD,



the Point K will coincide with C, and CK will become equal to nothing; whence after the Stroke the Body A will be at rest. But if FG is greater than



CD, the Point K will fall beyond C, and the Motion of A will become negative, or towards the contrary Parts, *viz.* from C towards K; and the Sum of the Motions towards the Parts G, will be as EG lessened by CK: for the Sum of two Quantities, whereof one is positive, the other negative, is their difference. But because $FC = KD$, let $EF - CK$ be added to them both, and it will be $EF + FG - CK$: that is, $EG - CK = KD + EF - CK$, that is, $EF + CD$: whence the Sum of the Motions towards the same Parts, which is here the Difference of the Motions towards the contrary Parts, before and after the Stroke, remains the same. Q. E. D.

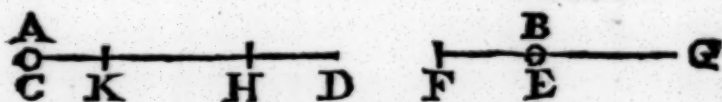
Cor. AFTER the same manner, if more Bodies moving towards the same Parts strike against one another, the Sum of their Motions will not be altered.

THEOR.

THEOR. XIX.

If two Bodies moving towards the contrary Parts, do directly strike against each other; the Sum of their Motions towards the same Parts (which is the Difference of their Motions towards contrary Parts) before and after the Stroke, will always remain the same towards the same Parts.

LET the Body A move from C towards D, whose Motion let be expressed by CD; but let B be moved towards the contrary Part, namely from E to F,

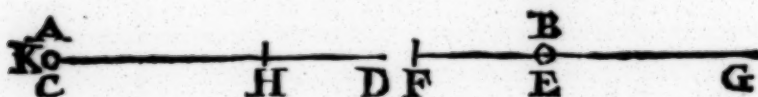


with a Motion as EF: let DH be put equal to EF; and CH, which is the Difference of the Motions towards the contrary Parts, will be as the Sum of the Motions towards G: I say, the same CH will be as the Sum of the Motions towards the same Part G after the Stroke. For let the Motion of the Body B after the Stroke be towards the Part G, and let it be represented by the right Line EG; the Force therefore of Impulse impressed on the Body B towards the Part G, will be equivalent to the Sum of the Motions EF, EG, and will be represented by the right Line FG: for by that Force is destroyed the Motion as EF, towards the Part F, and a new one as EG is impressed towards the contrary Part G. But since the Force of Impulse acts equally on both Bodies towards the contrary Parts, if DK is made equal to FG, it will represent the Force exercised on the Body A towards the Part contrary to its Motion; so that if the Motion as DK is subtracted from the Motion as CD, there will remain CK as the true Motion of the Body A towards the Part G.

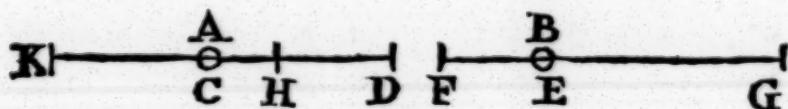
Now

Now since DK is equal to FG, and DH equal to FE, it will be DK, lessened by DH, that is, KH equal to FG, lessened by FE, that is, EG: and therefore since KH is equal to EG, KH will be as the Motion of the Body B after the Stroke; but CK is as the Motion of the Body A, so that CK, KH, that is, CH will be the Sum of the Motions in both Bodies towards the Part G. *Q. E. D.*

If FG is equal to CD, the Point K will fall on C, and then the Motion of A will become equal to



nothing; that is, the Body A after the Stroke will be at rest, and CH will be equal to EG. But if FG is greater than CD, the Point K will fall be-



yond C towards the other Part, and the Motion of the Body A will be from C towards K; but (by reason FG is equal to DK, and FE equal to DH) KH is equal to EG: and therefore if from both there be taken CK, it will be CH equal to the right Line EG, lessened by CK. But CH was as the Sum of the Motions towards the Part G before the Stroke, and it is EG lessened by CK, as the Sum of the Motions towards the same Part; namely, as the Difference of the Motions towards the contrary Parts after the Stroke. Wherefore the Sum of the Motions towards the same Part will remain the same before and after the Stroke.

THESE two last Theorems are both at once thus well expressed by Sir *Isaac Newton*.

THE

THE Quantity of Motion, that is gathered by taking the Sum of the Motions made towards the same Part, and the Difference of those made towards the contrary Parts, is not altered by the Action of Bodies amongst themselves.

LECTURE XIII.

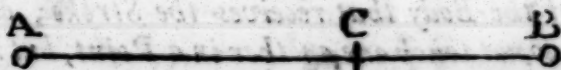
Second Definitions.

I.  HE Center of Gravity of any Body is a Point placed within that Body, through which if a Plane passes any how, the Segments of the heavy Body that are on each side, as it were ballanced upon that Plane, will equiponderate.

HENCE, if a Body is suspended by its Center of Gravity, whatever Situation it is put into, it will retain it: viz. since the Parts of the Body every where about the Center consist of equal Momenta, or have equal Propensities to Motion.

II. WE call the common Center of Gravity of two Bodies, a Point so situated in the right Line joining their Centers, that the Distances of the Bodies from that Point, are in a reciprocal Ratio of the Bodies.

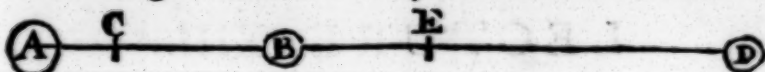
LET there be two Bodies A, B, whose Centers of Gravity let be joined by the right Line AB; which let be so divided in C, that AC be to BC as the



Body B, that is, the Matter of B is to the Body A, or Matter in A; that Point C is called the common
M Center

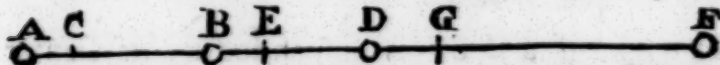
Center of Gravity of the Bodies A and B: namely, because if those Bodies are turned about that Point, at the same Distances from it, they will retain any given Situation, (as was demonstrated in Theor. 11.)

III. IN like manner, if there are three Bodies A, B, D, and C be the Center of Gravity of two of them, A and B, and the right Line CD be so divided in E, that CE



is to DE as the Weight of the Body D to the Weight of the two Bodies A and B together, that Point E is called the common Center of Gravity of these three Bodies; about which also if the Bodies are turned, they will retain any given Situation.

IV. AFTER the same manner, if there are four Bodies A, B, D, F, and E is the common Center of Gravi-



ty of the three A, B, D; the Point G, which so divides the right Line EF, that EG is to GF as the Weight of the Body F to the Weight of the Bodies A, B, D, taken together, is called the common Center of Gravity of these four Bodies.

AND after the same manner the common Center of Gravity of five or more Bodies may be determined.

V. ONE Body is said to strike or impinge directly on another, when the right Line, in which it moves, drawn through the Center of Gravity of the impinging Body and the Point of Contact, is perpendicular to the Superficies of the Body that receives the Stroke; or, if the Bodies do not touch one another in a Point, but in a Line or a Superficies, when that Line is perpendicular to this Line or Superficies.

VI.

VI. BUT it is said to strike or impinge obliquely or indirectly on the other, when the before-mentioned right Line is not perpendicular to the Superficies of the Body that receives the Stroke.

VII. I CALL that a perfectly hard Body, which does not yield in the least to a Stroke; that is, which does not lose its Figure for a Moment.

VIII. THAT is a soft Body, which so yields to a Stroke, as to lose its first Figure, and never to endeavour to recover it again.

IX. THAT is an elastic Body, which yields indeed for a little while to a Stroke, yet of its own accord does recover its first Figure.

X. AN elastick Force is that Force, whereby a Body obliged to quit its Figure, recovers it again.

XI. A PERFECTLY elastick Body is such a one, as recovers its first Figure with the same Forces whereby it was obliged to quit it.

THEOR. XX.

IF two or more Bodies are carried by an equable Motion towards the same or contrary Parts, their common Center of Gravity before their mutual Concurrence will either be at rest, or will move uniformly in a right Line.

The first Case. LET the Bodies A and B tend towards contrary Parts, whose common Center of Gra-



vity let be C. By reason of the equal Quantity of Motion in both Bodies, the Velocity of the Body A will be to the Velocity of the Body B, as the Body B to the Body A; that is, (from the Nature of the Center of Gravity) as AC to BC: whence, the Spaces passed over in the same Time being proportionable to the Velocities, whilst the moving Body A

runs through the Length AC , the Length BC will be run through by the moving Body B ; so that the Bodies will meet in the Point C , and in that Point will be their Center of Gravity at the time of their Concourse: but it was in the same Point before the Concourse, so that it remained in the same place.

It may be shewn, after the same manner, that if the Bodies receded from the Point C with equal Motions, their Center of Gravity would have been at rest.

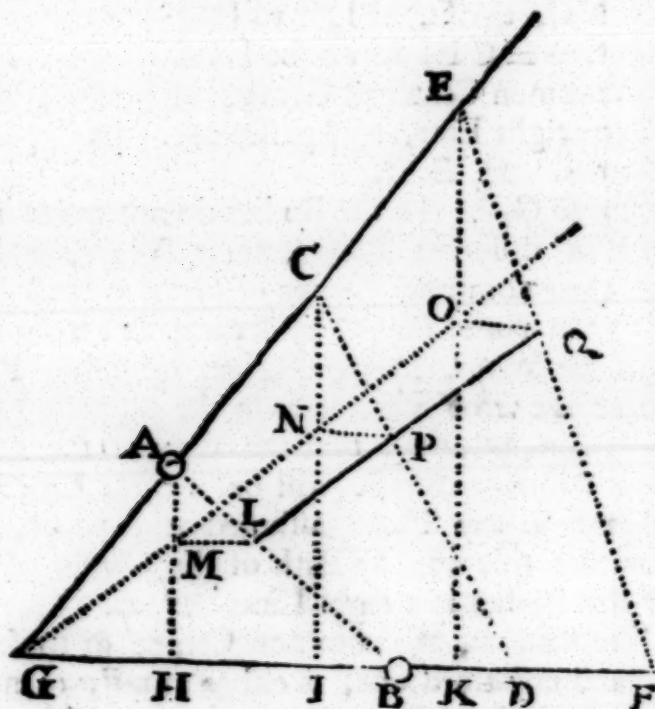
The second Case. If Bodies are carried in the same right Line towards the same Part, or with unequal Motions towards contrary Parts, their common Center of Gravity will be always found in the same right Line. For since the Bodies do uniformly and directly recede from one another, or approach one another, their Distance from one another will be uniformly increased or diminished; and therefore the Bodies will uniformly recede from, or approach to any Point, dividing the aforesaid Distance in a given Ratio. The Distance therefore of the Bodies from their common Center of Gravity will be uniformly increased or diminished; which cannot be in the aforesaid Cases, unless that Center is either at rest, (as in the first Case) or moves uniformly, as in the present Case.

The third Case. LET the Bodies A and B move in the right Lines AC , BD ; and let the Spaces AC , CE , passed over by the Body A in equal Times, be equal; and the Spaces BD , DF , passed over by the Body B in the same Times, be also equal. Let the right Lines AC , BD , meet in G ; and let it be as AC to BD so is AG to GH , and join AH ; to which through C and E let CI , EK , be drawn parallel: it will be AC to HI as AG to GH , that is, as AC to BD ; wherefore HI is $= BD$, and therefore $HB = ID$. In like manner, CE is to IK , as AG

to

Lect. 13. *Natural Philosophy.* 165

to GH, or AC to BD, that is, as CE to DF; wherefore IK is = DF, whence KF = ID = HB. Let L be the common Center of Gravity of the Bodies, when they are placed in the Points A and B; draw LM parallel to BD; the right Lines AB, AH, will be cut similarly: join GM, and let it be produced; this will cut the Parallels to AH in the



Points N and O; viz. in the same Ratio as AH or AB is cut. Draw through N and O, parallel to BD, the right Lines NP, OQ; these will cut CD, EF, in the same Ratio as CI, EK, are cut, that is, in the same Ratio as AB is cut in L: but L is the common Center of Gravity of the Bodies when they are found in A and B; wherefore P will be their Center, when they are in the Points C and D, and Q when in E and F. Besides, ML is to HB as AM to AH, or as CN to CI, or as NP to ID:

M 3

but

but HB and ID are equal; wherefore ML and NP will be equal: in like manner, NP and OQ will be equal. Since therefore the right Lines ML , NP , OQ , are equal and parallel, the right Line drawn through L , and parallel to MO , will pass through the Points P and Q , and therefore the Center of Gravity will be always placed in the right Line LQ . Besides, (by reason of the Parallels) AC is to CE as MN to NO , that is, as LP to PQ ; wherefore (by reason AC is $= CE$) it will be $LP = PQ$. Therefore the common Center of Gravity of Bodies is always in the same right Line, and passes over equal Spaces in equal Times. *Q. E. D.*

The fourth Case. IF the Bodies do not move in any one, but in different Planes, their Ways, and the Way of their common Center of Gravity must be reduced to the same Plane, by letting fall from every Point of the Ways, Perpendiculars upon any Plane; and (after the same manner as in the preceding Case) it will be demonstrated, that the Way of the Center of Gravity thus reduced, will be a right Line: and since this is in any Plane assumed at pleasure, it is necessary that this way or Path of the Center of Gravity of the Bodies is a right Line. *Q. E. D.*

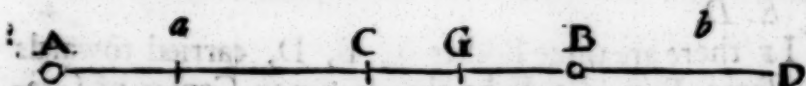
IN like manner, the common Center of these two Bodies, and any third one, is either at rest, or moves uniformly in a right Line, by reason the Distance of the common Center of Gravity of the two Bodies, and the Center of the third, is divided by it in a given Ratio. And after the same manner the common Center of these three Bodies, and a fourth, either is at rest, or moves in a right Line, by reason the Distance betwixt the common Center of the three, and the Center of the fourth Body, is divided by it always in the same Ratio; and so of any other number of Bodies. *Q. E. D.*

THEOR.

THEOR. XXI.

If two Bodies, whether equal or unequal, are carried towards the same Part, with Celerities whether equal or unequal, the Sum of the Motions in both Bodies will be equal to the Motion that would arise, if both the Bodies were carried with the Celerity of their common Center of Gravity.

LET the two Bodies be A and B, whose common Center of Gravity let be C, and let both the Bodies be carried towards D; I say, the Sum of the Motions in both Bodies will be equal to the Motion

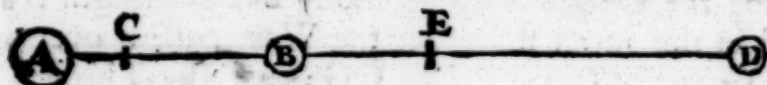


that would be produced, if both Bodies were carried towards D with the Celerity of their Center of Gravity C. For let the Body A in any given Time describe the Length A a , the Body B the Length B b , and in the mean while let the Way passed over by the Center of Gravity C be C G; and [by Theor. 6.] the Lengths A a , B b , C G, described in the same Time, will represent the Celerities of the Body A, the Body B, and the common Center of Gravity C respectively; but [by Corol. Theor. 3.] the Quantity of Motion in any Body is as a Rectangle contained under the Quantity of Matter and the Celerity, so that the Motion in the Body A will be as $A \times A a$; and in the Body B as $B \times B b$; and the sum of the Motions will be as the Sums of these Rectangles, viz. as $A \times A a + B \times B b$. But [by the Def. of the Center of Gravity of Bodies] B C is to A C as A to B; and as A to B, so also [by the same Definition] is $b G$ to $a G$: wherefore B C will be to A C as $b G$ to $a G$; whence [by 19 El. 5.] B C is to A C, that is, A to B, as B C — $b G$ to A C — $a G$; that is, as C G

— Bb to Aa — CG . So that [by 16 *El.* 6.] $A \times Aa$ — $A \times CG$ will be equal to $B \times CG$ — $B \times Bb$; and therefore $A \times Aa + B \times Bb$ will be equal to $A \times CG + B \times CG$: but the two Rectangles $A \times Aa$ and $B \times Bb$ are (as has been said) as the Sums of the Motions in both the Bodies: and the two Rectangles under A and CG , and under B and CG , will be as the Sum of the Motions that would arise, if both the Bodies were carried with the Celerity CG of the Center of Gravity: whence the Sum of the Motions in both Bodies is equal to the Motion that would be produced, if both Bodies were carried with the Celerity of their common Center of Gravity.

Q. E. D.

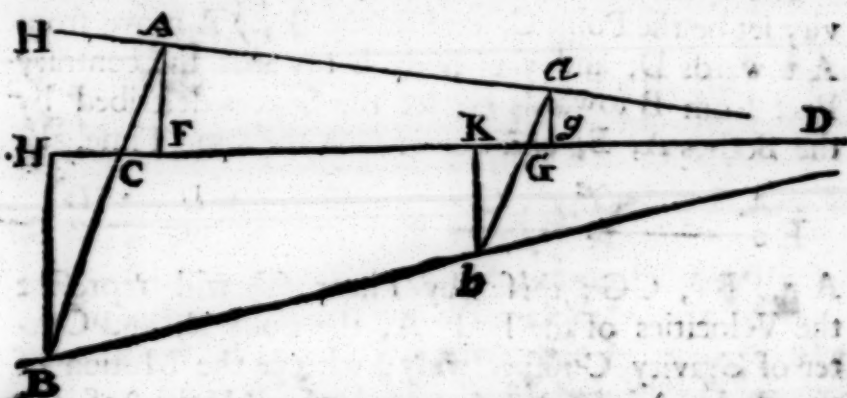
If there are three Bodies A , B , D , carried towards the same Part, let E be the common Center of Gravity of these three Bodies; the Sum of the Motions in the three Bodies will be equal to the Motion arising from the same Bodies carried with the Velocity



of the Point E . For let C be the common Center of Gravity of any two Bodies A and B , the Motion in these two Bodies (by what has been demonstrated before) will be equal to the Motion that would arise, if the Bodies coalesced in one were carried with the Velocity of the Point C ; but also the Sum of the Motions (*viz.* the Motion of the Bodies so coalescing, and the Motion of the third Body D) will be equal to the Motion which would happen, if the Body coalescing out of the two was moved together with the third Body D , with the Celerity of the Point E ; whence the Theorem holds also in this Case.

THE Demonstration is the same, if the Bodies do not move in the same right Line, but in Parallels, or also

also in Lines any how inclined. But in this Case it must be observed, that the Celerity of the Bodies, wherewith they are carried towards the same Part with the Center of Gravity, is not to be reckoned by the Way which they really pass over, but only by the Way wherein they move according to the Direction of the Center of Gravity. For example, if two Bodies A and B are carried in the right Lines Aa, Bb, and CG is the Line described by the common Center of Gravity, whilst the Bodies pass over the Lengths Aa, Bb, and from the Points A, a, B, b, let fall the Perpendiculars AF, ag, BH, bK upon the right Line CG. Now the Spaces which the Bodies pass over according to the Direction of the



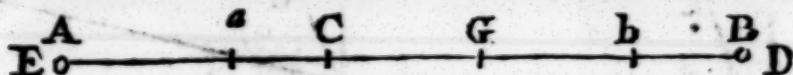
Point C, are not Aa, Bb, which are the absolute Spaces described by them; but the Space according to which the Body A is moved towards the Part D, must be computed in the right Line FD, by the Length Fg; for it moves so much, and no more, according to the Direction of the Point C. In like manner, the Space according to which the Body B is moved towards the Part D, is HK; and by that Space, its Progress in the right Line HD is to be reckoned: so that the Celerities of the Bodies wherewith they are carried towards the same Part, are as the

the right Lines Fg , HK ; besides, A is to B as BC to AC , or (by reason of the equi-angular Triangles ACF , BCH) as HC to FC : whence the Demonstration will proceed as in the first Case.

THEOR. XXII.

If two Bodies are carried towards contrary Parts, the Difference of the Motions towards the contrary Parts, or, which is the same thing, the Sum of the Motions towards the same Part, will be equal to the Motion that would be produced, if both the Bodies were carried towards the same Part with the Celerity of the common Center of Gravity.

LET the Bodies be A and B , whose Center of Gravity let be the Point C , and let the Body A move from A towards D , and the Body B towards the contrary Part from B towards E ; let the Spaces described by the Bodies A , B , and Center C in the same Time, be



Aa , Bb , CG : these [by Theor. 6.] will represent the Velocities of the Body A , the Body B , and Center of Gravity C respectively; whence the Motion of the Body A is as $A \times Aa$, and the Motion of the Body B as $B \times Bb$, so that the Difference of the Motions will be $A \times Aa - B \times Bb$. Moreover, from the Nature of the Center of Gravity, BC is to AC as A to B ; and as A to B , so will be bG to aG : wherefore it will be as BC to AC , so bG to aG . So that [by 19 *El.* 5.] BC will be to AC , that is, A to B as $BC - bG$ to $AC - aG$; that is, A will be to B as $Bb + CB$ to $Aa - CG$, wherefore [by 16 *El.* 6.] the Rectangle under A and $Aa - CG$ will be equal to the Rectangle under B , and $Bb + CG$; that is, $A \times Aa - A \times CG = B \times Bb + B \times CG$:
1
whence

Leſt. 13. Natural Philoſophy.

171

whence it will be $A \times Aa - B \times Bb = A \times CG + B \times CG$; but $A \times Aa - B \times Bb$ is (as was ſaid before) the Difference of the Motions towards the contrary Parts, or the Sum of the Motions towards the ſame, and $A \times CG + B \times CG$ is the Motion emerging, if both the Bodies ſhould be carried with the Velocity of their common Center of Gravity, whence the Propoſition is manifeſt.

Cor. 1. IF the Difference of the Motions towards the contrary Parts is equal to nothing, that is, if in both the Bodies the Quantities of Motion are equal, the common Center of Gravity in this Caſe will be at Reſt.

Cor. 2. IF there are ſeveral Bodies, either all carried towards the ſame Parts, or ſome towards the contrary Parts; the Sum of the Motions of all towards the ſame Parts will be the ſame, as if all were carried towards the ſame Part with the Velocity of the common Center of Gravity of them all.

Cor. 3. THE Motion therefore of many Bodies is to be reckoned from the Motion of the Center of Gravity, and their System will proceed forwards, or go backwards, aſcend or deſcend as much, as their common Center of Gravity proceeds forwards, or goes backwards, aſcends, or deſcends.

THEOR. XXIII.

IF Bodies impinge on one another, or do all any way upon each other, the State of their common Center of Gravity of being at reſt, or moving uniformly in a right Line, will not be thereby changed.

IF the Bodies impinge on one another, [by Theor. 19.] the Sum of the Motions towards the ſame Part will be the ſame before and after the Impulſe; but [by Theor. 21, and 22.] the Sum of the Motions before and after the Impulſe is the ſame, as if all the
Bodies

Bodies were carried with the Velocity of the common Center of Gravity, and towards the same Part with it. Wherefore, since the same Bodies have the Sums of their Motions before and after the Impulse equal to each other, and also equal to the Motion arising from all carried together with the Velocity of the common Center of Gravity: it is manifest, that the Velocity of the common Center of Gravity before and after the Impulse will be the same. *Q. E. D.*

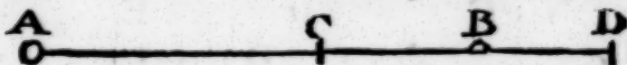
WE have hitherto delivered some general Laws, serving to determine the Motions of Bodies of all kinds; we shall now proceed to give some particular Rules relating to their meeting each other, *viz.* whereby each Body, after the Concourse, and mutually striking against one another, do continue their Motions, and towards what Parts they tend, and with what Velocity. But by reason of the different Structure of Bodies, *viz.* as some are endued with an elastick Force, whilst others want that Force, the Rules of their Concourse will be different according to the various Sorts of Bodies: and tho' perhaps there is no Body that is either perfectly hard, or perfectly soft, or perfectly elastick, (for haply all Bodies contain something of all these in themselves) yet it does not hinder, but that we may, by an Abstraction of the Mind, separate those Qualities, and consider a Body as only endued with one of these Qualities: and the Motions of Bodies will come the nearer to the Rules hereafter given, the more the Bodies themselves partake of those Qualities and Conditions.

WE here suppose the Bodies to be separated from all others, that their Motions are neither hindered or promoted by such as may lie round about them.

THEOR. XXIV.

IF a hard or soft Body directly impinges or strikes upon another hard or soft Body, whether the Body that receives the Stroke, is either at rest, or moves slower towards the same Part with the other, or lastly towards the contrary Parts, and the Motions are unequal, both Bodies after the Stroke will move together conjointly with the common Center of Gravity.

LET the Body A impinge on the Body B, which is either at Rest, or moves slower towards the same Part, or is carried towards the contrary Parts with a less



Motion; I say, that both the Bodies after the Impulse, will move with the same Celerity, conjointly with the common Center of Gravity. For since the Body B is not hindered by the circumjacent Bodies, (by the second Law) it will move by the Force impressed on it by the Body A, towards the Parts, wherein is the Direction of that Force; but it will also be moved conjointly with the Body A: For it cannot move slower, by reason the Body A follows immediately behind it; and it cannot move faster, because by Hypothesis there is given no other impelling Cause of its Motion besides the Body A; since we suppose all others, as an elastick Force, and an ambient Fluid, to have here no Influence: so that, after the Concourse, both the Bodies and the common Center of Gravity will move conjointly together. *Q. E. D.*

Cor. **IF** the Bodies are supposed to concur in D, since the Velocities of moving Bodies are as the Spaces described in the same Time, the Velocities of the Body A, the Body B, and the Center of Gravity

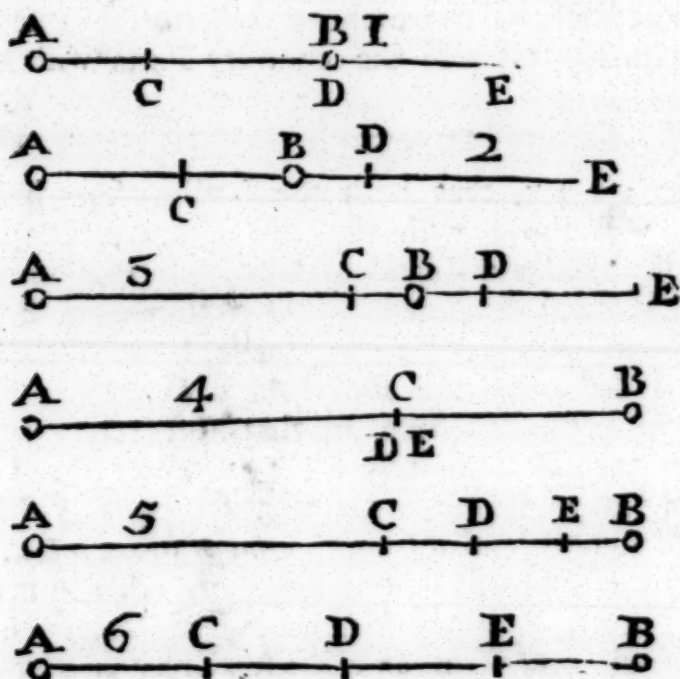
C,

C, will be before the Concourse, as the right Lines AD, BD, CD respectively; for these Lengths are passed over in the same Time.

PROB. II.

To determine the Motions of hard or soft Bodies after their direct Impact or Stroke.

WE shall construct all the Cases of this Problem at once. Let therefore A and B be two Bodies, whose Center of Gravity let be C, and let us sup-



pose the Bodies to meet in D, the Celerities of the Body A, of the Body B, and of the common Center of the Gravity C, before the Stroke, will be [by the preceding Corol.] as the right Lines AD, BD, and CD respectively. Let now DE be made equal to DC, this will represent the Velocity of the Bodies after

after their Concourse; that is, the Velocity of the Body A before the Impulse will be to its Velocity after the Impulse as A D to D E: and the Velocity of the Body before the Stroke, will be to its Velocity after the Stroke, as B D to D E: for [by Theor. 19.] the Bodies A and B after the Impulse move on together with the Center of Gravity; but [by Theor. 18.] the Celerity of the Center of Gravity will be the same both before and after the Impulse, and always towards the same Part. Wherefore if C D represents its Velocity before the Impulse, D E equal to C D will express its Velocity after the Impulse; so that D E will also represent the Celerity of the Bodies A and B, which after the Impulse move together with the Center C. Q. E. D.

Cor. 1. If the Body B is at rest, the Point D coincides with B, as in the first Figure; and because B is to A, as A C to B C, or D E, by Composition it will be $A + B$ to A, as A B or A D to D E; that is, the Velocity of the Body A before the Stroke is to its Velocity after the Stroke, as the Sum of the Bodies to the impinging Body A.

Example 1. If A is equal to the quiescent Body B, it will be $A + B$ to A as 2 to 1; so that the Velocity of the impinging Body will be double its Velocity after the Concourse.

Example 2. If A is to B as 1 to 9, it will be $A + B$ to A as 10 to 1; so that the Velocity after the Impulse will be only the tenth part of the Velocity before the Impulse.

Example 3. If B is a Body that exceeds A infinitely, the Velocity of the Body A, after the Impulse, will be infinitely small, that is, none at all; for in that Case, A in respect of $A + B$ vanishes, and therefore the Velocity of the Body A after the Concourse likewise vanishes: that is, if the Body A impinges on a firm and immoveable Obstacle, after the Stroke it will be at rest.

Ex.

Example 4. IF the Body B is equal to A, and moves slower in the same Direction, D E or C D will

$$\text{be} = \frac{AB}{2} + BD = \frac{AB + 2 BD}{2} = \frac{AD + BD}{2};$$

that is, the Velocity after the Impulse will be half the Sum of the first Velocities.

Example 5. IF the Bodies tend towards the same Parts with equal Motions, the Point D will coincide with C, as was demonstrated in Theor. 20. and C D, D E, will be equal to nothing, that is, both the Bodies, will be at rest, after their Concourse.

Cor. 2. HENCE the Law of the *Cartesians* is demonstrated to be false, whereby they contend that there is always preserved the same Quantity of Motion in the Universe: for Bodies which are not elastick, meeting each other from contrary Parts with the same Motions, mutually destroy each other's Motions.

Example 6. IF equal Bodies tend towards contrary Parts with unequal Motions, it will be D E or C D,

$$= CB - BD = \frac{AB}{2} - BD = \frac{AB - 2 BD}{2} = \frac{AD - BD}{2};$$

that is, the Velocity after the Impulse will be half the Difference of the first Velocities.

ALL these things easily flow from the Construction above; but since in Practise there is occasion for a Calculation, a general Solution of this Problem may be thus discovered by Calculation.

LET the Velocity of the Body A be called C, and the Velocity of the Body B be c ; and if the Bodies are moved according to the same Direction, the Sum of the Motions in both towards the same Part will be $AC + Bc$: but if they are moved towards contrary Parts, the Sum of their Motions towards the same Part will be $AC - Bc$; but [by Theor. 19.]
in

in all Bodies, the Sum of their Motions towards the same Part before and after the Impulse is the same: wherefore the Motion of the Bodies after the Impulse will be either $AC + Bc$ or $AC - Bc$, accordingly as the Bodies did tend towards the same or contrary Parts. There is therefore given the Momentum of the Bodies carried with the same Velocity, whence [by what has been said in *Lect. X.*] their Velocity will be likewise known; namely, if their Momentum is divided by the Bodies themselves, the Quotient will exhibit their Velocities, viz. $\frac{AC + Bc}{A + B}$ or $\frac{AC - Bc}{A + B}$; and if B is at rest, that is, if c be put equal to nothing, the Velocity of the Bodies will be $\frac{AC}{A + B}$.

Cor. 3. SINCE the Velocity of the Body A before the Impulse was as AD, and after the Impulse its Velocity was as CD, the Velocity lost will be AC, and therefore the Motion lost by the Stroke $A \times AC$.

THEOR. XXV.

IF a moving Body strikes directly against another either in motion or at rest; the Magnitude of the Stroke is proportionable to the Momentum that is lost at the Concourse by the more powerful Body, whichever of them it is.

FOR if we suppose the more powerful Body, (whichever it is) or if they have equal Momenta, either of the Bodies, to be that which gives the Stroke, and the other that which receives it; the Magnitude of the Stroke will be equivalent to the Force impressed by the first on the other: wherefore that Force impressed departs from the percutient Body, [by the third Law] so that the Motion lost in the percutient Body will be proportionable to

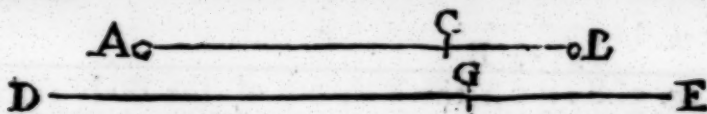
the Force impressed on the other Body, and consequently to the Magnitude of the Stroke. *Q. E. D.*

Cor. WHERE the Momenta are equal that depart from the percutient Bodies, there the Magnitudes of the Strokes will be equal.

THEOR. XXVI.

If a given Body directly impinges on another given Body at rest, the Magnitude of the Stroke will be always proportionable to the Velocity of the impinging Body.

LET the given Body A impinge on another given Body B that is at rest, with a Velocity which may be expressed by A B. Afterwards let the same Body A impinge on the same quiescent Body B, with ano-

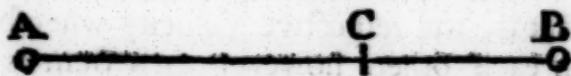


ther Velocity D E; that is, let A B be to D E as the former Velocity to the latter, and let the Distances of the Bodies be A B, D E: for whatever is the Distance at the beginning of Motion, it is the same thing in respect to the Magnitude of the Stroke; and let the common Center of Gravity in the first Situation be C, in the second G. Since the Body A is moved with the Velocity A B, its Velocity after the Concourse will be C B; and since its Motion before the Impulse was $A \times A B$, its Motion after the Impulse will be $A \times C B$; and the Motion lost will be $A \times A C$. After the same manner, if the Body is moved with the Velocity D E, the Motion lost will be $A \times D G$; and consequently, the Magnitude of the Stroke with the Velocity A B will be to the Magnitude of the Stroke with the Velocity D E, as $A \times A C$ to $A \times D G$, or as A C to D G: but because it is A C to B C as B to A, it will be

A C

AC to $AC + BC$, that is, AB, as B to $A + B$, and in like manner, it will be B to $A + B$ as DG to DE. Wherefore it will be AC to AB, as DG to DE, whence by permutation it will be AC to DG as AB to DE; that is, the Magnitude of the Stroke with the Velocity AB, will be to the Magnitude of the Stroke with the Velocity DE, as the Velocity AB to the Velocity DE. *Q. E. D.*

Cor. If the Body A should rush on B, the Motion lost would be as $A \times AC$; but if B should impinge on A with the same Velocity, the Motion lost would

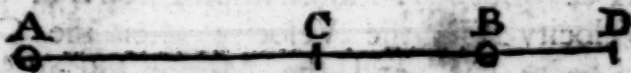


be as $B \times BC$; but because A is to B as BC to AC, it will be $A \times AC = B \times BC$: so that the Quantity of Motion lost by the Stroke, will be the same, whether B impinges on A with a given Celerity, or A with the same Velocity strikes against B; and consequently in both Cases the Magnitude of the Stroke will be the same.

THEOR. XVII.

If one Body should impinge on another Body moving slower, but in the same right Line and towards the same Part, the Magnitude of the Stroke would be the same, as if the antecedent Body was at rest, and the following Body was carried towards it with a Velocity equal to the Difference of their Velocities.

LET the two Bodies carried towards the same Part be A and B, whose common Center of Gravity let be C, and let the Point of their Concourse be D;



from what has been delivered above, the Velocities of the Bodies before the Impulse will be as the right

N 2

Lines

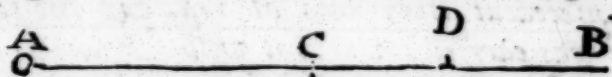
Lines AD, BD, and consequently the Difference of their Velocities will be as AB: but the Velocity of both the Bodies after their Concourse will be represented by CD, and consequently the Motion lost in the Body A will be $A \times AC$. But if the Body A should impinge with the Velocity AB on the quiescent Body B, its Velocity after the Concourse would be CB, and its Motion lost $A \times AC$: whence since in both Cases the same Quantity of Motion is lost in the percutient Body, the Magnitude of the Stroke will be also the same. *Q. E. D.*

Cor. IF the Difference of Velocities remains the same, that is, the respective Velocity whereby Bodies approach each other; however their Sum is increased or diminished, the Magnitude of the Stroke will be always the same.

THEOR. XXVIII.

IF two Bodies meet each other with contrary Motions, the Magnitude of the Stroke will be the same, as if one of them was at rest, and the other impinged on it with the Sum of their Velocities.

LET A and B be two Bodies carried towards contrary Parts, whose common Center of Gravity let be C, and let D be the Point where they concur; it's plain that the Velocities of the Bodies A and B are as the right Lines AD, BD; and consequently the



Sum of the Velocities will be represented by AB; but CD expresses their Velocity after the Impulse, and therefore the Motion lost in the Body A will be $A \times AC$. But if A should impinge on B at rest, with the Velocity AB; the Velocity after the Concourse would be as CB, and the Motion lost as $A \times AC$. Since therefore in both Cases the same Quantity of Motion is lost, the Magnitude of the Stroke will be the same. *Q. E. D.*

Cor.

Lect. 13. *Natural Philosophy.* 181

Cor. 1. IF therefore the Sum of the Velocities remains the same; that is, the respective Velocity of the Bodies A and B, wherewith they approach each other, whatever is the Difference of Velocities, or after what manner soever that Velocity is shared amongst the meeting Bodies, the Magnitude of the Stroke will be always the same.

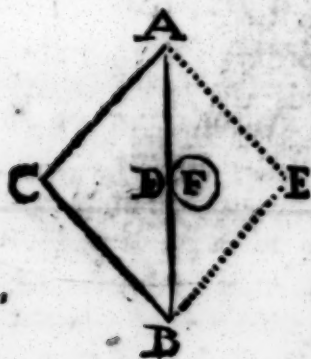
Cor. 2. THE Magnitude therefore of the Stroke in given Bodies is always proportionable to their respective Velocities.

Cor. 3. THE Motions of Bodies inclosed in a Space are the same amongst one another, whether that Space is at rest, or moves uniformly and directly on; for the Differences of Velocities, wherewith Bodies tend towards the same Part, and the Sums, wherewith they tend towards the contrary Parts, are the same, whether the Space, in which they are inclosed, is at rest, or moves uniformly forwards: and therefore the Magnitudes of the Strokes being always proportionable to these, will be the same in both Cases. Hence all the Motions within a Ship are performed in the same manner, whether it is at rest, or is moved uniformly forwards. So also the Phenomena of Projectiles and Percussions do all happen just the same amongst us placed upon the Earth, whether they are all together with the Earth in one common Motion, or there is no such common Motion, and the Earth is at rest; so that the Objections that are wont to be brought from the Inequality that ought to be in Projectiles not mov'd by the same Force, accordingly as they are made towards the East or towards the West, and from the unequal Percussions made by a Bullet hurled by a Cannon, as the Explosion is performed towards this or that part, and the like, prove nothing on either side, whether to establish the Rest or the Motion of the Earth.

LECTURE XIV.

IF there were no such thing as *Elasticity*, the Laws, which we have proposed in the preceding Lecture, relating to the Percussions of hard Bodies, would perfectly agree to all Bodies, and all Bodies after the Impulse would tend conjointly towards the same Parts, towards which, before the Percussion, the more powerful Body did tend, that is, the Body whose Momentum was the greatest; and with the Velocity that we determined in the before mentioned Laws. But since there are indeed very few Bodies, wherein there is not some degree of Elasticity, (for soft Clay, Wax, and the like Bodies, contain in themselves some Particles of Air, which gives them some Elastick Virtue) it happens by that Elastick Force, that Bodies after their Impulse do not move conjointly together, but fly from one another, and are moved with a different Velocity, sometimes towards the same, and sometimes towards the contrary Parts. But in order to understand the Modus and Cause of this Rebounding in Bodies, we will illustrate the thing by an Example.

LET *AB* be a String extended over a Plane, but at some Distance from it, whose Extremities *AB* let be firmly fixed, and the String itself very much stretched: Now if the String be drawn by its middle Point *D*, its Extremities remaining immoveable, into the Situation *ACB*, so that its Point *D* shall be in *C*, and then be let go, the String will not remain

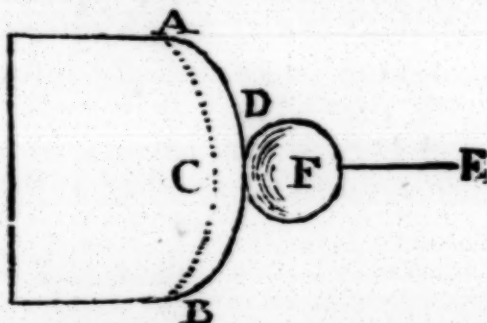


remain in the Situation A C B, but with a great Force will endeavour to restore itself to its former Situation: and since by the continual Action of the Elastick Force a sufficiently swift Motion is raised in the String, it will happen that when its Motion perseveres towards the same Part, till the Elastick or Restoring Force, constantly resisting this farther Motion, and at length equalling it, shall destroy it, and urge or compel the String towards the Parts C; so that when it shall again arrive into the Situation A D B, it will have the same Force of moving farther towards C, as it had at first of tending towards the Parts E: and so by moving backwards and forwards, it will make continual Vibrations.

LET us now suppose the Body F to strike against the String A B, the String will be then compelled to quit its Situation through the Force exerted on it by the Body F; and its Point D, on which the Body F gave the Stroke, will be moved, together with F, towards C: which Motion will continue even till the restoring Power of the String, which is contrary to the Motion of F, shall be equal to that Motion; which as soon as it is, all the Motion towards C will be destroyed. But this elastick Force acting farther, will bring back the String, which therefore will compel the Body F, and move it along with it with the same Velocity. But (by reason we suppose the Tension of the String to be very great) the String will restore itself with the same Force, wherewith it was first inflected: now the Force wherewith it was inflected, was equivalent to the Momentum of the impinging Body, (for that was all spent in bending the String) so that the String by acting on the Body F with such a Force, will restore to it the same Quantity of Motion, as was spent in bending it; and

therefore the Body F will be sent back with the same Velocity that it came forwards with, and so a Reflection will be occasioned.

LET us now put, instead of a String, any elastick Body A B, which at first we shall suppose fixed and immoveable, and let its Superficies A D B be pressed



inwards by the Force of the Body F striking against it. As soon as the compressing Force, that is, the Motion of the Body F shall cease, the springing Body by its innate Force will restore itself to its former Figure, and by that Force will compel the Body F towards E; and if both the Bodies are perfectly elastick, the restoring Force of the Spring will be equivalent to the Force that compresses it, that is, to the Momentum of the Body F: and therefore it acting on the Body F with this Force, it will compel that Body to return back with the same Velocity which it had at first. But if the Body A D B C is not fixed, but in such a state as its Motion is not hindered by any other Body, then the elastick Force will equally exert itself on both the Bodies, and it will produce equal Changes of Motions: for if the Body A D B compels the Body F towards the Part E, that again will be equally compelled by the Body F towards the contrary Part; and therefore the Bodies will rebound from each other. And so we have demonstrated after what manner it happens, that Bodies after the Impulse

pulse do not either rest, or move conjointly together, but by rebounding from one another, they move with different Velocities, sometimes towards the contrary Parts, and sometimes towards the same.

THE *Cartesians*, who were ignorant of the Force of Elasticity to reflect Bodies, having given a quite different Cause of Reflection: for they say, that Motion is not contrary to Motion, but its Direction to its Direction; and therefore one Body striking against another, is reflected, because the Motion of the Body that gives the Stroke cannot be destroyed, since according to them nothing can be contrary to Motion: but since the Direction of one opposes the Direction of another, they will have it, that the striking Body after the Impulse must be reflected towards the contrary Parts, the Quantity of Motion remaining the same in the Body that gives the Stroke, and in that which received it.

BUT it is easy to shew, that this Notion is neither agreeable to Reason or Experience: for since the Momentum or Quantity of Motion constitutes that Force or Energy wherewith the moving Body tends according to its Direction, if two Bodies directly meet each other, the Forces impressed according to their contrary Courses, will be contrary: so that if they are equal, they will mutually destroy each other; if unequal, the Motion that is of the less efficacy, will be destroyed. Besides, one Body striking against a greater Body at rest, or moving slower towards the same Parts, is reflected; but this cannot be done only by reason the Direction is contrary to the Direction: for if the Body B strikes against another greater Body A, that is either at rest, or moves slower towards the same Parts; since all the Force that is found in both
the



the Bodies, tends towards C, that Force can never direct the Motion in both the Bodies towards the contrary Parts. For [by the second Law] all Motion is performed according to the right Line, wherein the Force is impressed; but [by Hypothesis] all the Force is impressed according to the right Line B C, from B towards C: wherefore if the Reflection of Motion was only made by the Force that is innate in Bodies, without a new Force, there would be caused a Motion in a Course that is contrary to that wherein the Force is impressed; which cannot be. Therefore that Reflection does not arise from the Force first impressed, but from an Elastick Force, with which both the Bodies are endued, and which equally acting both ways, compels the Bodies to rebound from one another.

BESIDES, if Motion was not contrary to Motion, it would be much easier to direct a Body that is once in motion, towards the contrary Parts, than to stop it quite: for in the first Case, the Motion of the Body reflecting on one's Hand is not received, but only turned towards the contrary Parts; but in the latter Case, all that Motion is employed on the resisting Body, which yet is contrary to manifest Experience. Lastly, if nothing was contrary to Motion, where-ever any Body hit against any Obstacle, there will always be a Reflection, which is nevertheless contrary to Observation: for Lead, Clay, Wax, and other Bodies that have scarce any Elasticity in them, if they fall on the Pavement, are not reflected; yet when Balls made of Wool or Feathers, Ivory, Marble, or Glass, and other the like Bodies, which are very elastick, are thrown against the same Pavement, they strongly rebound: That Reflection therefore does not proceed from the Motion that is common to all Bodies, but from an Elasticity which is only peculiar to reflecting Bodies. Which was to be shewn.

But

BUT perhaps the *Cartesians* may ask, how we come to know that Balls of Ivory, Glass, and Marble, and other reflecting Bodies, which seem to be exceeding hard, are endued with an Elasticity: I answer, That they are elastick, may be hence concluded, That when they are struck, they yield a Sound, which arises from the Vibrations of the Body that receives the Stroke, after the same manner as a stretched String, by its Vibrations causes Undulations in the Air; and therefore it is not in the least to be doubted, but that these Bodies are endued with some Spring. And this Argument indeed makes it probable that these Bodies have an elastick Force, but there is another Argument, whereby this matter may be demonstratively proved.

FOR let there be two Balls either of Ivory or Glass, and if their Figures were perfectly spherical, they would touch one another in one only and that an indivisible Point; but this cannot be done by any human Art: yet they may be made so nearly spherical, that they shall touch one another in a physical Point, that is, in the least visible part. Now if the Superficies of one of the Balls is stained with Ink (or with any Colour that is easily wiped off) and another Body impinges on it at rest, it will appear from the Experiment, that not only a physical Point of the Ball that strikes against the other, will, after the Impulse be stained with the Colour of that other Ball, but a sufficiently large Part thereof; and this could not be, unless their Superficies were changed by the Force of the Stroke: but after the Reflection, we find that both the Balls have recovered their first Figure. Wherefore the Balls have an elastick Force, whereby they are able to recover their former Figure that was alter'd by the Stroke. *Q. E. D.*

Now follow the Rules of Motion observed by Elastick Bodies.

THEOR.

THEOR. XXIX.

If two perfectly Elastick Bodies impinge on one another, their relative Velocity will be the same before and after the Impulse; that is, the perfectly Elastick Bodies will recede from each other after the Stroke, with the same Celerity, wherewith they first approached one another.

FOR [by Cor. Theor. 27.] the compressing Force, on the Magnitude of the Stroke in given Bodies, arises from the relative Velocity of the Bodies, and is proportionable to it; and [by Def. 11.] perfectly Elastick Bodies restore themselves to their former Figure by the same Force, whereby they were compressed: that is, the restoring Force is equal to the compressing Force, and therefore is equivalent to the Force wherewith the Bodies approach each other before the Impulse. But Bodies are compelled to separate from one another by this restoring Force, whence this Force acting on the same Bodies, will produce a relative Velocity equal to that which they had at first, or it will make the Bodies to recede from one another with the same Velocity wherewith they at first approached each other. *Q. E. D.*

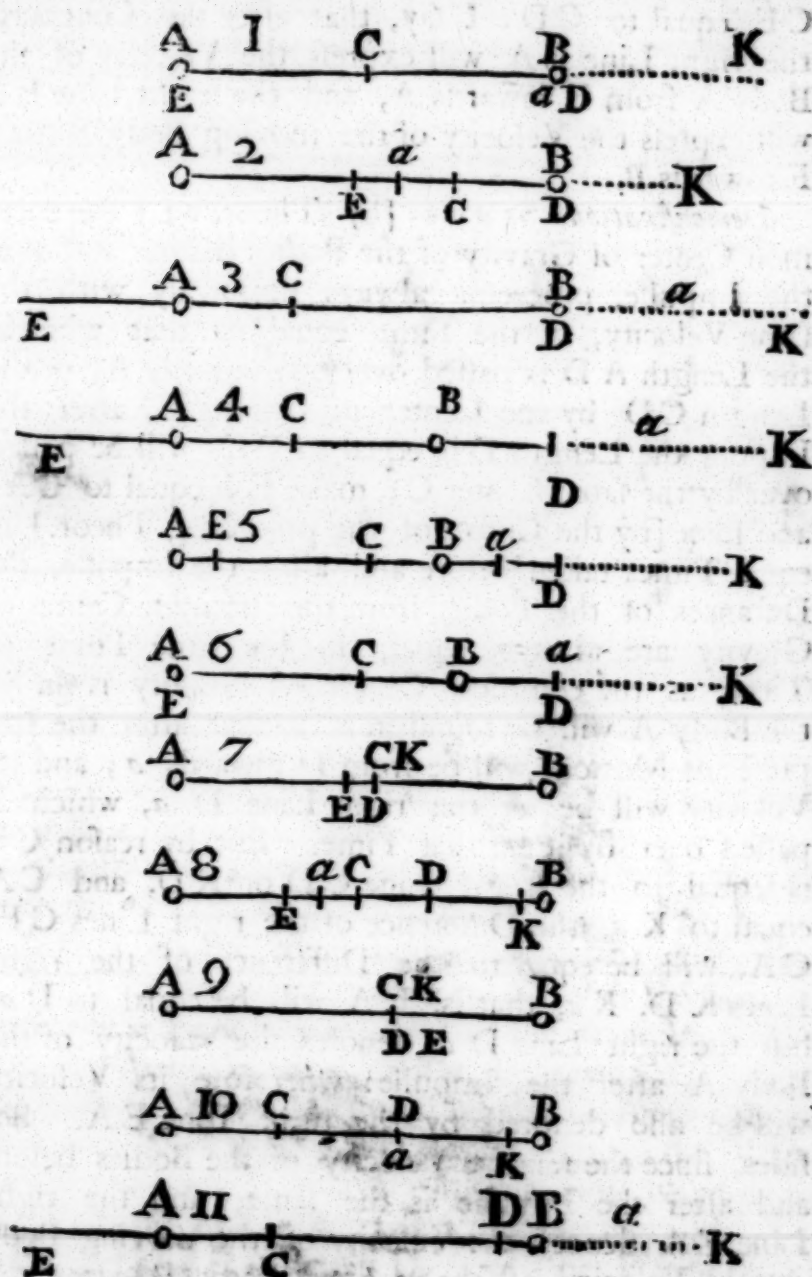
Cor. THEREFORE in equal Times taken before and after the Impulse, the Distance of the Bodies from one another will be equal; and consequently in the same Times, the Distances of the Bodies from the common Center of Gravity will be also equal.

FROM this Corollary, the Rules of Motion in the Congress of perfectly Elastick Bodies may be easily discovered; which we shall therefore do in the following Problem.

PROBL. III.

To determine the Rules of Congress in Bodies that are perfectly Elastick, and which do impinge directly on one another.

WE shall give at once the Construction of all the Cases of this Problem. Let A and B be two perfectly



fectly Elastick Bodies, whose common Center of Gravity let be C, and let the Bodies meet in D; make CE equal to CD: I say, that after the Concourse the right Line EA will express the Velocity of the Body A from E towards A, and the right Line EB will express the Velocity of the moving Body B from E towards B.

Demonstration. SINCE [by Theor. 23.] the common Center of Gravity of the Bodies before and after the Impulse, proceeds always uniformly with the same Velocity, in the Time equal to that wherein the Length AD is passed over by the Body A, or the Length CD by the Center of Gravity C, after the Impulse the Length DK equal to DC, will be passed over by the same Center C; make Ka equal to CA; and since [by the Corol. of the preceding Theor.] in equal Times taken before and after the Impulse, the Distances of the Bodies from the common Center of Gravity are always equal, in the same Point of Time, as the common Center of Gravity is in K, the Body A will be found in a, so that after the Impulse its Motion will be from D towards a; and its Velocity will be as the right Line Da, which is passed over by it in that Time. But by reason CE is equal to the right Line CD or KD, and CA equal to Ka, the Difference of the right Lines CE, CA, will be equal to the Difference of the right Lines KD, Ka, that is, EA will be equal to Da; but the right Line Da denotes the Velocity of the Body A after the Impulse: wherefore its Velocity will be also denoted by the right Line EA. Besides, since the relative Velocity of the Bodies before and after the Impulse is the same, and the right Line EA denotes the Velocity of the moving Body A, the Velocity of the moving Body B, after the Impulse must be necessarily denoted by the right Line EB; namely, from E towards B. Q. E. D.

Cor. 1. IF the Body B is at reſt, the Point D will coincide with B, as in the firſt three Figures: and becauſe B is to A as AC to CB, it will be by Compoſition $B + A$ to A as AB to CB: whence by doubling the Conſequents, it will be $B + A$ to 2 A, as AB to 2 CB or EB; that is, as the Aggregate of the Bodies to twice the impinging Body, ſo is the Celerity of the impinging Body before the Impulſe, to the Celerity of the Body that is firſt at reſt after the Contact.

Cor. 2. AND therefore if A and B are equal, it will be $A + B = 2 A$; whence EB, the Celerity of the Body B, after the Contact will be equal to AB the Celerity of the Body A before the Contact, and conſequently the Point E coinciding with the Point A, AE the Velocity of the moving Body A after the Impulſe will be equal to nothing: which may be alſo thus eaſily ſhewn: By reaſon of the Bodies A and B being equal, it will be $AC = CB = CD = CE$, wherefore the Point E coincides with A, and conſequently the moving Body A, after the Impulſe, will be at reſt, and the Body B after the Impulſe will move with the Celerity EB or AB. If therefore an elaltick Body impinges on another equal to it and at reſt, after the Contact the impinging Body will be at reſt, and that which was at reſt will be moved with the Celerity of the firſt Body.

Cor. 3. IF the equal Bodies A and B are carried towards the ſame Part (as in Fig. 4.) after the Contact they will be alſo carried towards the ſame Part with interchanged Velocities. For by reaſon CE is = CD, and $AC = CB$, it will be $CE - AC$, that is, $EA = CD - CB$ or BD; and therefore the Velocity of the Body A after the Impulſe will be equal to the Velocity of the Body B before the Impulſe. Beſides, becauſe EA is = BD, it will be $EB = AD$; and conſequently the Velocity of the Body B after the Contact will be equal to the Velocity that the Body A had before the Concourse,

Cor.

Cor. 4. IF the equal Bodies A and B are carried towards the contrary Parts, (as in Fig. 8.) they will after the Impulse recede towards the contrary Parts, with interchanged Velocities. For by reason $AC = CB$ and $CE = CD$, it will be $AC - CE$, that is, $AE = CB - CD$ or BD ; and therefore the Velocity of the Body A, after the Impulse, will be equal to the Velocity the Body B had before the Impulse. Besides, by reason $EA = BD$, it will be $AD = EB$; but AD was the Velocity of the Body A before the Concourse, and EB is the Velocity of the Body B after the Concourse, whence the Corollary is evident.

BUT since Calculation is always used in Practice, we shall here deliver a Method, whereby the Celerities of Elastick Bodies after the Impulse may be investigated, and brought to Numbers; and it would indeed be easy, after the manner of the foregoing Corollaries, to reduce to Numbers all the particular Cases from the general Construction; but a general Calculation may be thus more readily discovered.

LET us in the first place suppose the Bodies A and B to be moved towards the same Part; and let C be the Velocity of the following Body A, but c the Velocity of the preceding Body B: whence the



relative Velocity of the Bodies will be $C - c$, and the Sum of the Motions towards the same Part $AC + Bc$.

Let the Velocity of the Body A after the Impulse towards the same Part, as before, be called x ; and because the relative Velocity of the Bodies both before and after the Concourse is the same, the Velocity of the Body B will be $x + C - c$: for the relative Velocity of the Bodies is equal to the Excess of the Velocity, whereby the Velocity of the swifter Body exceeds that of the

the slower, so that the Excess ought to be $C - c$; but since the Velocity of the Body A is x , its Motion towards the Part D will be Ax ; and since the Velocity of the Body B is $x + C - c$, its Motion towards the same Part will be $Bx + BC - Bc$; and the Sum of these Motions will be equal to the Sum of the former Motions, that is, it will be $Ax + Bx + BC - Bc = AC + Bc$: whence by reducing this Equation, it will be $Ax + Bx = AC - BC + 2Bc$; and $x = \frac{AC - BC + 2Bc}{A + B}$ = the

Velocity of the Body A. Moreover, the Velocity of the Body B is $= x + C - c = \frac{AC - BC + 2Bc}{A + B} + C - c = \frac{AC - BC + 2Bc + AC + BC - Ac - Bc}{A + B} = \frac{2AC - Ac + Bc}{A + B}$.

If BC is greater than $AC + 2Bc$, x or $\frac{AC - BC + 2Bc}{A + B}$ will be a negative Quantity; so that

the Velocity of the Body A in that case will be towards the contrary part, and its Motion towards D will be negative. If the Body B is at rest, that is, if $c = 0$, the Velocity of the Body A after the Impulse will be $+$ $\frac{AC - BC}{A + B}$ forwards or backwards, as

the Sign $+$ or $-$ shall prevail.

If the Bodies A and B carried towards the contrary Parts with the Celerities C and c , do directly impinge on each other, their Motion towards the same Part will be $AC - Bc$, and the relative Velocities of the Bodies will be $C + c$. Now let x be the Velocity of the Body A after the Impulse; its Motion towards the same Part as before, will be Ax , and the Velocity of the Body B will be $x + C + c$, (for the relative Velocity of Bodies is not altered by

the Stroke) and the Motion in the Body B towards D will be $Bx + BC + Bc$; whence the Sum of the Motions towards the same Part will be $Ax + Bx + BC + Bc$, which [by Theor. 14.] will be $= AC - Bc$: so that it will be $Ax + Bx = AC - BC - 2Bc$, and $x = \frac{AC - BC - 2Bc}{A + B}$, and the Ve-

locity of the Body B will be $\frac{AC - BC - 2Bc}{A + B} + C + c$
 $= \frac{AC - BC - 2Bc + AC + Ac + BC + Bc}{A + B} = \frac{2AC + Ac - Bc}{A + B}$.

If $BC + 2Bc$ is greater than AC , the Motion of the Body A will be backwards; namely, towards the contrary Part: in which case x or $\frac{AC - BC - 2Bc}{A + B}$

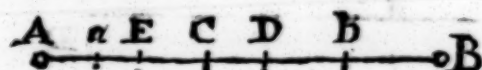
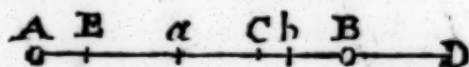
will be a negative Quantity.

THE first, as I know of, who gave the true Laws of Motion in hard Bodies, was Dr. *John Wallis*, the famous *Savilian* Professor of Geometry in this University, which he did in the *Philosophical Transactions* N^o 43. where he also shew'd the true Cause of Reflections in other Bodies, and proved that they proceeded from their Elasticity. Not long after, the famous Sir *Christopher Wren*, and *Monf. Huygens*, imparted to our *Royal Society* the Laws that are observed by perfectly Elastick Bodies, and gave exactly the same Construction, tho' each was ignorant of what the other had done. But since they published in the *Philosophical Transactions*, the Constructions and Laws of Motion without any Demonstration; we have thought fit to take thence their very elegant Construction, and to demonstrate it.

AFTER a not unlike Method is constructed the Problem in such Bodies as are indeed Elastick, but which do not restore themselves with a Force equal to that whereby they were compressed. For let there be any two Bodies A and B, whose common
 Center

Lect. 14. *Natural Philosophy.* 195

Center of Gravity let be C; let A C, B C, be so cut in a and b , that A C is to a C and B C to b C, as the



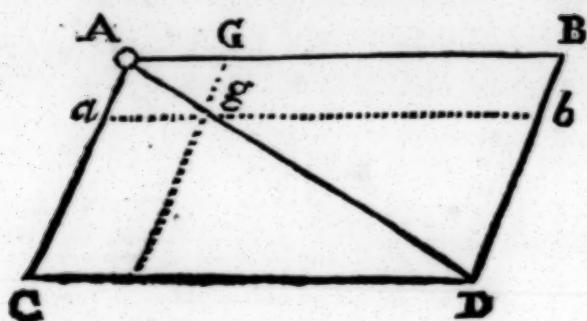
Force compressing the Elastick Body to the Force whereby it restores itself: and let C E be equal to C D, E a will be the Velocity of the Body A after the Impulse from E towards a , and E b will be the Velocity of the Body B from E towards D.

BUT if the restoring Force is equal to the compressing Force, the Point a will coincide with A, and the Construction returns to the former. The Demonstration is easy to any one that understands the preceding, nor is it necessary to add it here.

THEOR. XXX.

IF a Body A is moved uniformly in the right Line A B, and in the mean while that right Line A B is carried always parallel to itself, as also with an equable Motion according to a Direction that is parallel to A C; and let the Velocity of the Body A be to the Velocity of the Line A B as A B to A C, and let be compleated the Parallelogram A B D C, whose Diagonal let be A D: this will be the true Line described by the Body A in its Motion.

WHEN the Line A B shall come to the Situation ab , let g be the Place of the Body A; and because [by Theor. 6.] the Spaces described in the same Time are as the Velocities, ag the Length passed over by the moving Body A, will be to A a the Length passed over by the Line A B, as the Velocity of A to the Velocity of the right Line A B, that is [by Hypothe-



fis] as AB to AC : whence the Parallelogram aG will be similar to the Parallelogram CB , and consequently (by 24 *El.* 6.) the Point g will be placed in the Diagonal AD ; that is, the Body A will be always found in the right Line AD : so that this Line will be passed over by it. *Q. E. D.*

Cor. 1. THE Line AD will be described by the moving Body A in the same Time, wherein, without the Motion according to AC , it would pass over the Line AB ; or wherein, without the Motion according to AB , it would describe the right Line AC .

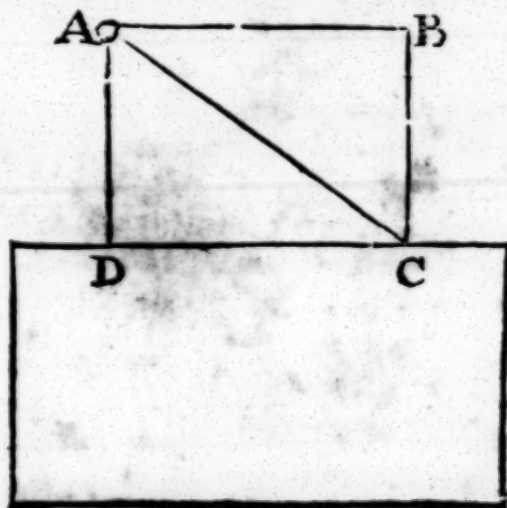
Cor. 2. SINCE the moving Body is therefore carried in the right Line AD , because beside its own proper Motion, it likewise partakes of the Motion of its Place, or of the right Line AB , and its Motion is compounded of both these; if any moveable Body has two Motions impressed on it at the same time, according to the Directions AB , AC ; and those Motions, or the Forces whereby they are produced, are as the right Lines AB , AC : AD will be the Line described by the moving Body, that receives the Motions impressed by these two Forces, and its Force, whereby it is carried in the right Line AD , will be to the Forces according to AB , AC , as the Diagonal AD to the Sides of the Parallelogram AB , AC .

Cor. 3. HENCE conversly, if the moving Body with a Force as AD passes over the right Line AD ,
the

the Motion will be the same, and according to the same Direction, as if at the beginning of the Motion it had been impelled by two Forces at once, each proportionable to the right Lines AB , AC , according to the Directions from A to B , and from A to C . And hence any Motion whatever, tho' simple in it self, may be consider'd as compounded of more Motions; and any Forces may be resolv'd into others, that are more in number, and act according to different Directions.

THEOR. XXXI.

If the Body A impinges obliquely on a firm Obstacle DC , the Energy of the Percussion, or Magnitude of the oblique Stroke, will be to the Magnitude of the Stroke that



the same Body would have produced, if it had impinged perpendicularly with the same Celerity, as the Sine of the Angle of Incidence ACD to the Radius.

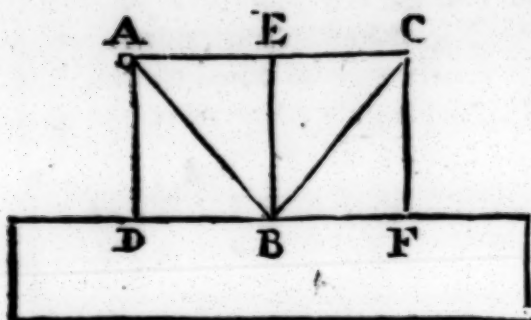
FROM A let fall on the Obstacle the Perpendicular AD , if the Superficies of the Obstacle is a Plane; or if a Curve, let the Perpendicular be let fall on the
 O 3 Plane

Plane touching the Obstacle in the Point of Incidence C, and compleat the Rectangle D B. Now [by Cor. 3. of the preceding Theor.] the Motion of the Body A being as A C in the right Line A C, will be equivalent to two Motions impressed at the same time according to the Directions A B, A D, which are to the Motions in A C as the right Lines A B, A D, to A C; but the Obstacle does no ways resist the Motion in the right Line A B; for since A B is parallel to D C, the Body moved in the right Line A B will never impinge on the Obstacle D C. The Force therefore wherewith it impinges on the Obstacle, is as the right Line A D: the Force therefore of the Body A in the right Line A C, is to the Force wherewith it impinges on the Obstacle, as A C to A D. But if it had impinged perpendicularly on the same with a Force as A C, the Magnitude of the Stroke would have been represented by A C, for the whole Motion would have been destroyed by the Obstacle: wherefore the Magnitude of the oblique Stroke will be to the Magnitude of the perpendicular Stroke as A D to A C; that is, A C being made the Radius, as the Sine of the Angle of Incidence to the Radius.

T H E O R. XXXII.

I F a perfectly Elastick Body impinges obliquely on a firm Obstacle, it will be so reflected from it, that the Angle of Reflection shall be equal to the Angle of Incidence.

LET the perfectly Elastick Body A fall obliquely on the firm Obstacle according to the Line A B; I say, that will with the same Celerity be so reflected in the right Line B C, that the Angle of Reflection C B F shall be equal to the Angle of Incidence A B D. Let the right Line A B express the Motion of the Body A in the Direction A B: [by Corol. 3. Theor.



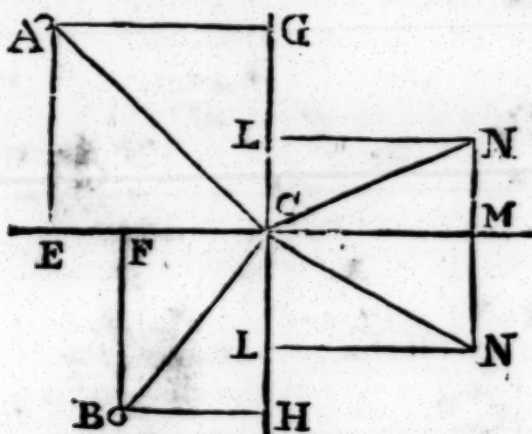
Theor. 30.] let this Motion be resolved into two others, according to the Directions, AE , AD , to which the Motion in AB is as AB to AE , AD : but since AE is parallel to the Superficies of the Obstacle, and AD perpendicular to it, or at least to the Plane touching the Obstacle in B ; that Force wherewith it impinges on the Obstacle, is only that which is as AD acting in a Direction that is perpendicular to the Obstacle. Now make BE equal and parallel to AD , and BF equal to DB or AE , and compleat the Rectangle EF , which will be in every thing similar and equal to the Rectangle DE . Since therefore the Motion as AE , according to the Direction parallel to the Obstacle is not destroyed by the Stroke, for the Obstacle does not oppose this Motion; after the Impulse at B , the Force remaining in the Body to move according to the Direction BF , will be as AE or BF : but from the Nature of Elasticity, a Body impinging on an Obstacle with a Force as EB according to the Direction EB , is reflected with the same Force according to the same Direction; the Motion therefore of the Body at the Point of Incidence B is compounded of the Motion as BF according to the Direction BF , and the Motion as BE according to the Direction BF . Wherefore [by Corol. 2. Theor. 30.] the Body will be moved in the right Line BC with a Force as BC : but by

reason AD , CF , are equal and parallel, and by reason DB , BF , and the Angles at D and F are equal, the Angle CBF will be equal to the Angle ABD , that is, the Angle of Reflection equal to the Angle of Incidence. *Q. E. D.*

PROBL. IV.

To determine the Motions of Bodies impinging obliquely after the Concourse.

LET any Bodies A and B be moved in the Lines AC , BC , inclined to one another, whose Lengths let respectively express the Velocities of the Bodies A , B : let EFC represent the Plane whereby the Bodies are touch'd in the Point of Concourse; on which from A and B let fall the Perpendiculars AE ,



BF , which may express the Velocities wherewith the Bodies approach each other. Compleat the Rectangles EG , FH . [By Cor. 3. Theor. 30.] the Motion of the Body A is resolved into two others, according to the Directions AG , AE ; to which the Motion in AC is as AC to AG , AE , respectively; in like manner, the Motion of the Body B is resolved into two other, according to the Directions BH ,
 BH ;

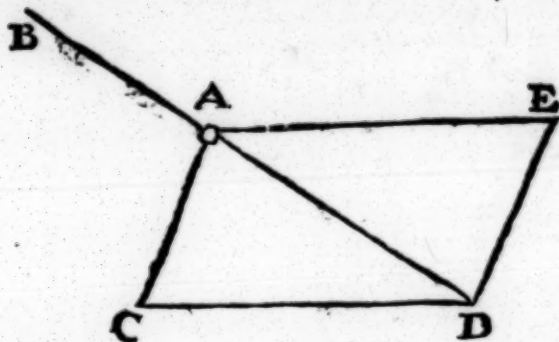
BH; to which the Motion in BC is as BC to BF, BH, respectively. But since AG, BH, are parallel, by the Velocities wherewith the Bodies are moved according to these Directions, they will not impinge on one another, so that the Motion according to these Directions will not be changed by the Impulse; the Velocities therefore wherewith the Bodies meet each other, are as AE or GC and BF or HC. The Motions therefore of the Bodies A, B, directly striking against each other with the Velocities GC, HC, will be determined, [by Probl. 2. if the Bodies are hard, or by Probl. 3. if elastick;] and let CL be the Velocity of the Body A from C towards L after the Impulse, which arises from the Velocities GC, HC. And since, as has been shewn, there remains in the Body a Force of moving according to the Direction parallel to AG, with a Velocity as AG, make CM equal to AG, and compleat the Rectangle LM: the Body A, after the Impulse, will move in its Diagonal CN with a Velocity as CN, as appears by Corol. 2. Theor. 30. and after the same manner will be determined the Motion of the Body B after the Impulse. Q. E. F.

THEOR. XXXIII.

If a moveable Body A is drawn by three Powers by the means of three Strings, or by any other manner whatever is sollicitated according to the Directions AB, AE, AC, so that these three Powers are equivalent to each other, that is, if any two of them destroy the Effect of the other, and the Body is moved by none of them; these Powers will have amongst themselves the same Ratio, as have the right Lines parallel to their Directions, and terminate by their mutual Concourse.

LET AD express the Power or Force wherewith the moveable Body A is sollicitated from A towards B; the

the Force equivalent or equal to this, and solliciting the Body A the contrary way from A towards D, will be also expressed by AD: but [by Cor. 3.

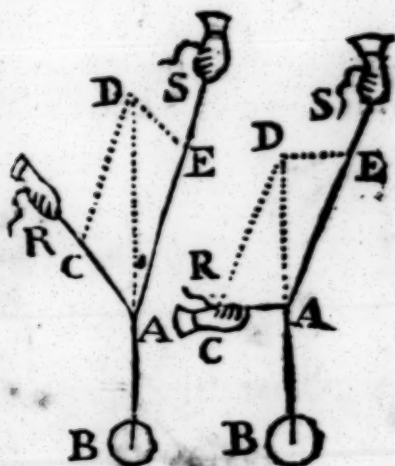


Theor. 30.] the Force impelling the Body from A towards D, will be equivalent to two Forces acting according to the Directions AC, AE, to which the former Force acting from A towards D, is as AD to AC, AE, or to AC, CD, respectively; and inversely the Forces acting according to the right Lines AC, AE, and equivalent together to the Force solliciting the Body from A towards D, ought to be to the same Force according to AD as AC and AE or CD to AD: wherefore likewise the Forces acting according to the right Lines AC, AE, and equivalent to the Force wherewith the Body is sollicitated from A towards P, and destroying its Effect, ought to be to the same, as AC, CD, to AD; that is, if the same moveable Body is sollicitated by three Powers equivalent to each other, according to the Directions AB, AC, AE, these three Powers will be as the right Lines AD, AC, AE, respectively. Q. E. D.

Cor. 1. SINCE in every Triangle the Sides are as the Sines of the opposite Angles, AC will be to CD as the Sine of the Angle ADC or DAE to the Sine of the Angle DAC: whence any two Powers will

will be amongst themselves reciprocally as the Sines of the Angles, which the Lines of their Directions contain with the Line of Direction of the third Power. Besides, AD is to AC as the Sine of the Angle C or AED to the Sine of the Angle CDA or DAE; and in like manner, the Power acting according to AB, is to the Power according to AE as the Sine of the Angle AED to the Sine of the Angle ADE or CAD.

Cor. 2. If two Powers R, S, sustain a Weight B, by the means of Strings drawing in the right Lines AR, AS, the Point A will be sollicitated by three Powers, whereof two act according to the Directions AR, AS, and the other is the Force of the Gravity of the Weight B, acting according to the right Line AB perpendicular to the Earth;



whence the Power R will be to the Force of Gravity as AC to AD, or as the Sine of the Angle DAE to the Sine of the Angle DEA or CAE; and the Power S will be to the Force of Gravity as EA to AD, or the Sine of the Angle CAD to the Sine of the Angle DEA or CAE; and the Power R will be to the Power S as the Sine of the Angle EAD to the Sine of the Angle CAD.

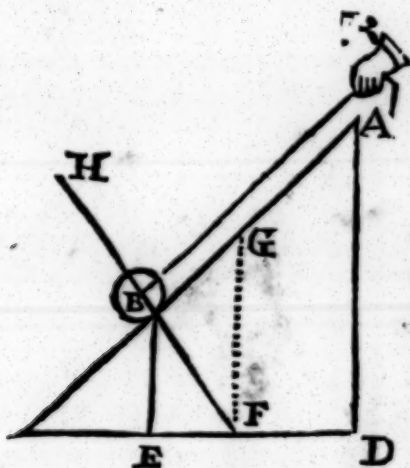
THIS Theorem, with its Corollaries, is the Foundation of the *New Mechanicks* published by M. Varignon; and from it likewise may immediately be deduced most of the mechanick Theorems contained in the famous Book of Borelli, *de Motu Animalium*, for by its help may the Force of the Muscles be determined.

THEOR.

THEOR. XXXIV.

IF an heavy Body B is incumbent on an inclined Plane, and is supported by a Power R, acting according to a Direction that is parallel to the Plane, so that it does not descend along that Plane; the Power R will be to the Weight of the Body B as the Sine of the Angle of Inclination to the Radius.

THOUGH the Point where the heavy Body rests against the Plane, draw AC perpendicular to the common Section of the Plane and the Horizon;



from any Point of which A, let fall on the Plane of the Horizon the Perpendicular AD, and join CD; ACD will be [by Def. 6. El. 11.] the Angle of Inclination of the Plane and the Horizon, whose Sine is AD, CA being made the Radius. Now I say, AC is to AD as the Weight of the Body B to the Power R. For the Body B is

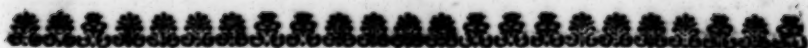
solicited by three Powers acting according to different Directions, and placed in *equilibrio*; whereof the first is the Force of Gravity acting according to the Direction BE perpendicular to CD; the second is the Power R, drawing the Body according to the Direction BR parallel to AC; but the place of the third Power is supplied by the Resistance of the Plane acting according to the Line BH perpendicular to it; for Re-action is always equal to Action, and is made towards the contrary Part: and since the Plane is pressed perpendicularly by the moveable Body,

Body, according to the Direction BH , and that contrary Endeavour is equivalent to the Power solliciting the Body according to BH ; and since these three Powers are *in equilibrio*, and the moveable Body is sustained by them, if FG is drawn parallel to EB , meeting the right Line AG in G , the Power R will be to the Force of Gravity as BG to FG , [by the preceding Theor.] but by reason the Triangle CFG is rectangular, and FB a Perpendicular to its Base CG , it is [by 8 *El.* 6.] as BG to FG , so is FG to GC , and as FG to GC , so [by 4 *El.* 6.] will be AD to AC : wherefore the Power R is to the Force of Gravity as AD to AC , or as the Sine of the Inclination of the Plane to the Radius. Any Power therefore may be able to sustain a heavy Body on an inclined Plane, provided the Power is to the Weight of the heavy Body as the Sine of the Inclination of the Plane to the Radius. *Q. E. D.*

Cor. 1. SINCE the Power R hinders the Descent of the heavy Body down the Plane AC , and is equivalent to its Momentum, whereby it endeavours to descend down the Plane; it follows that the Force of any heavy Body descending down an inclined Plane, is to the Force wherewith it endeavours to descend in the Perpendicular, as the Sine of Inclination of the Plane to the Radius.

Cor. 2. HENCE there may be assigned such an Inclination of a Plane, that any, the smallest Power, may be able to sustain or even raise upon that Plane any Weight, how great soever.





LECTURE XV.

Of the Descent of heavy Bodies on inclined Planes, and of the Motion of Pendulums.

HAVING finished what belongs to Motion in general, we shall now proceed to treat of those Motions that arise from given Forces; in explaining of which, and reviewing the Phenomena thence arising, consists the Business of the true Philosophy. That therefore we may begin with what is most simple, we shall in the first place consider that Force which is always directed toward the same Part uniformly, that is, every where with the same Tenour; and such is commonly supposed the Force of Gravity. For though it is certain that the Force of Gravity is not every where the same, but at different Distances from the Center of the Earth, is reciprocally as the Squares of such Distances; yet since the different Heights to which heavy Bodies can be thrown by us, are very small, in comparison of the vast Distance from the Center of the Earth, in so very little Difference of Heights, we may safely, and without any sensible Error, suppose the Force of Gravity to be every where the same.

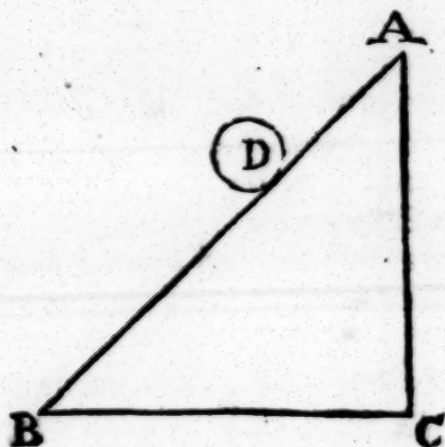
WE shall therefore in this place treat of the Motion of heavy Bodies: but we suppose this Motion to be performed, either in Planes inclining to the Horizon, or in curve Superficies, such as are spherical and cycloidal ones; or lastly, in free and unresisting Spaces: concerning all which, we shall give the following Theorems.

THE-

THEOR. XXXV.

THE Descent of a heavy Body upon any inclined Plane, is a Motion equably accelerated. And the Velocity that a heavy Body falling from Rest along an inclined Plane, acquires in any given Time, is to the Velocity acquired in the same Time, by a heavy Body falling perpendicularly, as the Height of the Plane to its Length.

LET AB be the inclined Plane on which the heavy Body D descends. [By Corol. 1. Theor. 34.] the Force wherewith a heavy Body endeavours to de-



scend on an inclined Plane, is to the absolute Force of Gravity, namely that whereby it would descend in the Perpendicular, in a constant Ratio, which is that of the Sine of the Inclination of the Plane to the Radius, or as the Height of the Plane to its Length; and therefore since the absolute Force of Gravity of the Body D remains the same, the Force likewise wherewith it endeavours to descend on the Plane AB, will remain the same. That Force therefore will always act on the heavy Body D in the same Tenour; so that being applied in the same manner, it will, by the second Law, always add equal Increments of Velocities,

ties, not otherwise than is done in heavy Bodies falling perpendicularly. The Descent therefore of heavy Bodies on an inclined Plane, is a Motion uniformly accelerated. *Q. E. D.*

MOREOVER, the Increments of the Velocities of heavy Bodies falling in the Perpendicular, and on an inclined Plane, which are produced in the same indefinitely small Time, are to one another as the Forces wherewith they are produced: but the Forces are in a constant Ratio, namely, as the Length of the Plane *AB* to its Height *AC*; wherefore the Increments of the Velocities thence arising, will be in the same Ratio. And consequentld, [by 12 *El.* 5.] the Sum of the Increments of one, will be to the Sum of the Increments of the other, in the same Ratio, that is, the Velocity of the heavy Body falling perpendicularly, is to the Velocity of a heavy Body descending in the mean time on an inclined Plane, as the Length of the Plane to its Height. *Q. E. D.*

Cor. 1. THE Velocities of a heavy Body falling on an inclined Plane, are as the Times wherein they are acquired.

Cor. 2. WHATEVER therefore we have demonstrated in Theor. 12. and its Corol. concerning Motion uniformly accelerated, the same will hold true in the Descent of heavy Bodies on inclined Planes; namely, the Space passed over in a given Time by a heavy Body falling on an inclined Plane, computing from the beginning of Motion, will be the half of that, which in the same Time may be passed over by a Body moving uniformly, with the Velocity acquired at the last. Also the Spaces passed over, computing from the beginning of the Motion, are in a duplicate Ratio of the Times or Celerities; and the Celerities and Times are in a subduplicate Ratio of the Spaces passed over.

Cor.

Cor. 3. HENCE also the Ascent of a heavy Body along any inclined Plane, is a Motion uniformly retarded, as it is in the Ascent of a Body in the Perpendicular, and it has just the same Properties.

SCHOLIUM.

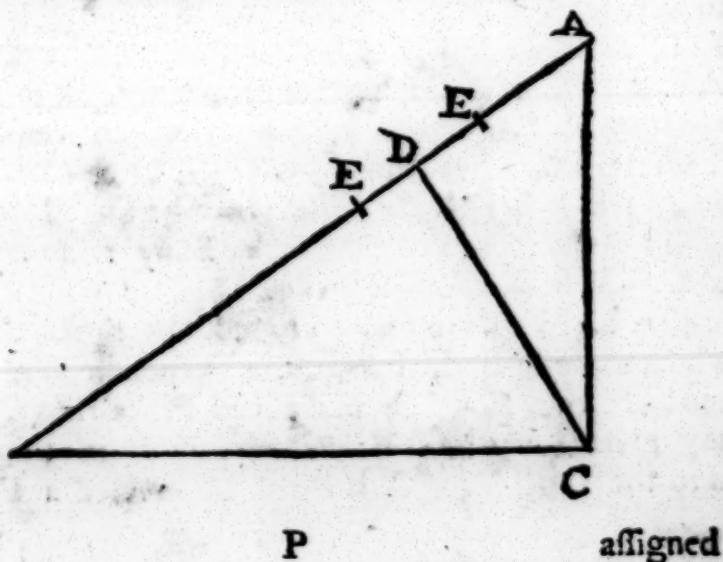
IF we have recourse to Experiments, we shall find all things answer to our Reasonings; and in Places that are not too much inclined, it will be easy to make an Experiment, since Motions that are not very swift may be exactly measured: But it is the contrary in the perpendicular Descent, where the Swiftneſs of the Motion gives not time to make accurate Observations.

WE here ſuppoſe the Planes to be exactly ſmooth, and the Motion on them to be hindered by no Ruggedneſs or Unevenneſs.

PROBL. V.

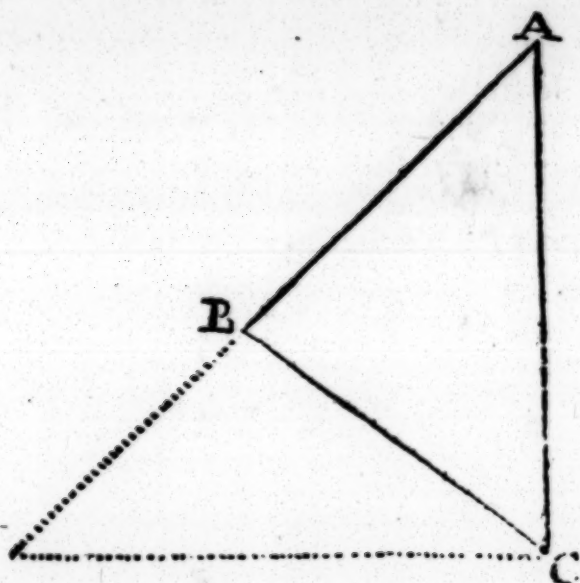
AN inclined Plane being given, to aſſign the Part of it which a heavy Body will paſs over, in the Time whiſt another heavy Body ſhall have fallen through a given Space in the Perpendicular.

LET AB be the inclined Plane on which the heavy Body deſcends from A; the Length is to be



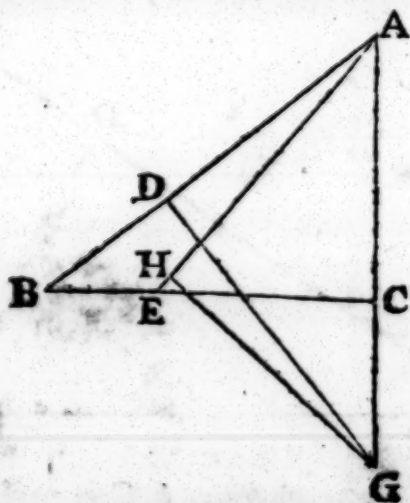
assigned in it, which will be passed over by the heavy Body falling along the inclined Plane; whilst in the mean while another heavy Body falling perpendicularly, shall have run over the Space AC . From the Point C let fall the Perpendicular CD on AB meeting the Plane in D ; then AD will be the Space passed over along the inclined Plane, in the Time wherein the heavy Body falls in the Perpendicular from A to C . For if AD is not, let AE be the Space described in the same Time that the heavy Body falls from A to C , which let be greater or less than AD . Draw the horizontal right Line CB . And because by Theor. 12. in the Time that the heavy Body falls from A to C , or from A to E , it may run over twice the Length of AC , with an uniform Velocity, and equal to that which it acquires from falling to C , [as by the Cor. of the preceding Theor.] in the same Time a Length double of AE may be run over, with the Velocity that is acquired in E ; the Velocity in C , by Theor. 6. will be to the Velocity acquired in E as twice AC to twice AE , or as AC to AE . But since AC , AE , are run over in the same Time, the Velocity in C [by the preceding Theor.] will be to the Velocity in E as AB to AC : wherefore AB will be to AC as AC to AE ; but [by 8 *EL* 6.] as AB to AC , so is AC to AD : wherefore AC will be to AE as AC to AD ; and consequently, AE will be equal to AD , a less to a greater, which cannot be. Therefore no other Space besides AD can be passed over by a heavy Body falling along the Plane AB , whilst in the mean time a heavy Body falls from A to C . Which was to be shewn.

Cor. 1. HENCE may be found the Space through which a heavy Body falls in the Perpendicular, whilst another heavy Body on an inclined Plane runs through any given Length AB : namely, if from the Point B to the right Line AB there be erected a Perpendicular



dicular BC, meeting the Perpendicular in C, then AC will be the Space sought.

Cor. 2. If there are two or more inclined Planes AB, AE; and there is given the Space AD in the Plane AB, passed over by a heavy Body in any Time;



the Space that will be run through in the same Time by a heavy Body on the Plane AE, will be found

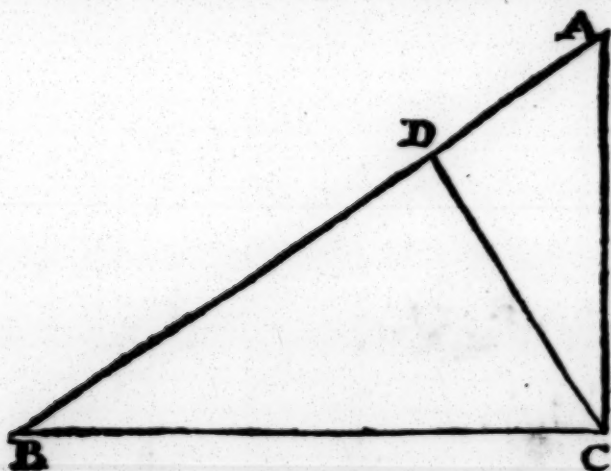
found by erecting from the Point D the Perpendicular DG, meeting with the Perpendicular in G: and by letting fall from G on A E the Perpendicular GH, meeting the Plane A E in H; then AH will be the Space sought: for both the Spaces A D, A H, will be passed over in the Time that the heavy Body descends in the Perpendicular from A to G.

Cor. 3. FROM the Demonstration of this Theorem it is manifest, that the Velocities acquired in the same Time by heavy Bodies falling in the Perpendicular and on inclined Planes, are as the Spaces passed over by those Bodies.

THEOR. XXXVI.

THE Time wherein the inclined Plane AB is passed over, is to the Time wherein the Perpendicular AC is run through, as AB the Length of the Plane, to AC the Length of the Perpendicular.

FROM C to AB let fall the Perpendicular CD; and the Time wherein AD is passed over, will be

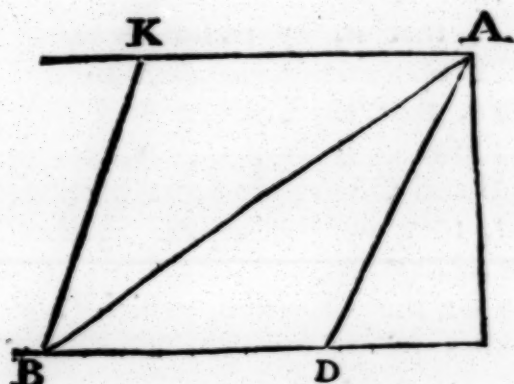


equal to the Time wherein AC is passed over. But the Time wherein AB is passed over, is to the Time wherein AC is passed over, in a subduplicate Ratio

Lect. 13. *Natural Philosophy.* 213

Ratio of AB to AD , [by Corol. 2. Theor. 35.] that is, by reason AB , AC , AD , are continual Proportionals, the Time wherein AB is passed over, is to the Time wherein AD or AC is passed over, as AB to AC . *Q. E. D.*

Cor. HENCE the Time wherein different Planes, AB , AD , BK , whose Height is the same, are passed over, are as the Lengths of the Planes: for the Time in AB is to the Time in AC , as AB to AC ;



and the Time in AC to the Time in AD , as AC to AD : wherefore by Equality the Time in AB will be to the Time in AD , as AB to AD .

THEOR. XXXVII.

THE Celerities of heavy Bodies, on any inclined Plane, and in the Perpendicular, are equal, where the heavy Bodies shall descend from the same Height to the same Horizontal right Line. See Fig. Theor. 36.

LET the inclined Plane be AB , and the Perpendicular AC . Draw the Horizontal right Line BC . I say, the Celerity acquired in the Point B , after the Descent through AB , will be equal to the Celerity acquired in the Point C , after the Fall through AC . From the Point C let fall the Perpendicular CD on

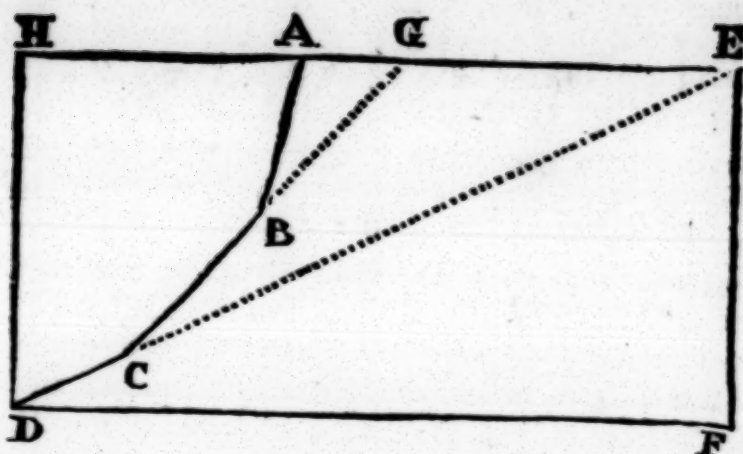
A B. Then A D will be the Space that is run over by the heavy Body falling in the Plane A B, in the Time that another heavy Body descends in the Perpendicular through A C: and [by Cor. 3. Probl. 5.] the Celerity in C is to the Celerity in D as A C to A D, or as A B to A C. But because the Celerities acquired in falling along the same Plane are in a subduplicate Ratio of the Lengths run over by the heavy Body, the Celerity in B will be to the Celerity in D, in a subduplicate Ratio of the Length A B to the Length A D; that is, by reason A B, A C, A D, are continual Proportionals, as A B to A C: but it has been shewn, that the Celerity in C is to the Celerity in D, also as A B to A C; wherefore since the Celerities in B and C have the same proportion to the Celerity in D, they will be equal. *Q. E. D. See Fig. Cor. of the preceding Theor.*

Cor. HENCE the Celerities, which are acquired by heavy Bodies falling from the same Height to the same Horizontal right Line, on Planes inclined after any manner, are equal amongst themselves; for both the Celerities, namely, that which is acquired in the Point B, after the Descent through A B or B K; and that which is acquired in the Point D, after the Descent through A D; is equal to the Celerity acquired in the Descent of the heavy Body from A to C.

THEOR. XXXVIII.

If from the same Height a moveable Body descends with an uninterrupted Motion through any Number of contiguous Planes A B, B C, C D, it will always at the conclusion acquire the same Velocity; namely, that which is equal to what is acquired by falling perpendicularly from the like Height.

THROUGH A and D draw the Horizontal right Lines H E, D F, and produce the Planes B C, C D, that

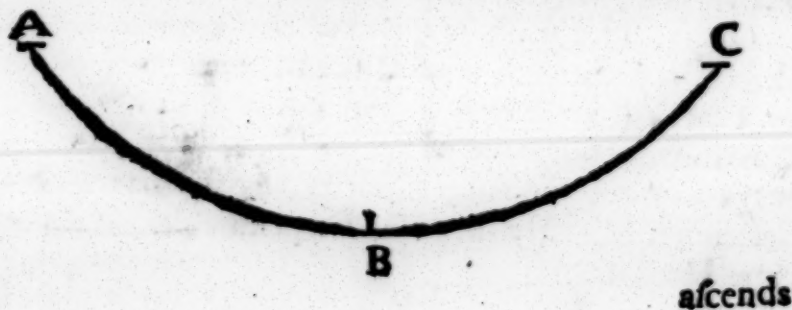


that they may meet with HE in the Points G and E. [By Corol. Theor. 37.] the same Celerity is acquired in the Point B, by descending through A B, as if the heavy Body had descended through B G: but we suppose the Flexure at the Point B, not to hinder the Motion of the heavy Body, but only to change its Direction; so that in the Point C the same Velocity will be acquired by descending through A B, B C, as if it had descended through G C. But by descending through G C, the same Celerity is acquired, as the heavy Body would have gotten by falling through E C; so that since the Flexure at C is not supposed to diminish the Velocity of the heavy Body, it will have the same Velocity in D, as if it had descended through the Plane E D, or the Perpendicular E F. *Q. E. D.*

Cor. 1. HENCE it appears, that a moveable Body descending through the Circumference of a Circle, or through any Curves, (for we may here consider Curves as compounded of infinite right Lines) always acquires the same Velocity, as if a heavy Body descended directly in the Perpendicular from the same Height.

Cor. 2. BUT if a heavy Body, after its Descent through A B, B C, C D, or through H D, should turn its Motion upwards; it would ascend to the same Height from whence it fell, through any inclined Planes: for since Gravity always acts with the same force in the same Plane, whether the Body ascends or descends, its Efficacy will be the same to diminish the Velocity in the Body in its Ascent, as to increase it in its Descent; therefore the Decrement of the Velocity in the Point C, whilst the moveable Body ascended from D to C, is as great, as was the Increment of the Velocity acquired in the Descent from C to D: and consequently the Velocity in C will be the same after the Ascent through C D, as it was before in the same Point, after the Descent through A B, B C. In like manner, the Velocity in B after the Ascent through C B is the same with the Velocity acquired in the Descent through A B or B G. So also Gravity will take as much from the Velocity of the Body in ascending through B A, as was acquired in the Descent through A B; and in the Points that are of an equal Height, the Velocity of the Body will be always the same, but the Velocity in the beginning of the Descent, namely, in the Point A, was nothing: so that by ascending, at that Point A all the Velocity is taken away; which Point therefore will be the Boundary to which the moveable Body by ascending will arrive.

Cor. 3. If a moveable Body descends through any Superficies A B to its lowest Point B, and afterwards

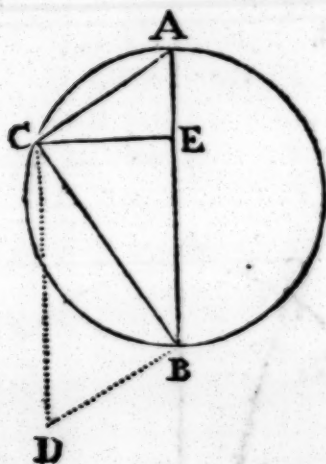


ascends by the Velocity acquired in the Fall, through the similar and equal Superficies BC; it will ascend and descend in equal Times through equal Spaces.

THEOR. XXXIX.

If from the highest Point A, or lowest B, of a Circle perpendicular to the Horizon, be drawn any two inclined Planes AC, BC, to the Circumference; the Times of the Descents through those Planes, will be equal to the Time wherein a heavy Body falls perpendicularly through the Diameter.

LET a heavy Body fall from A to C, on the Plane AC; I say, the Time of Descent through AC is equal to the Time of Descent through the Diameter AB. For the Angle ACB in the Semicircle is a right one, [by 31 *El.* 3.] whence since from the

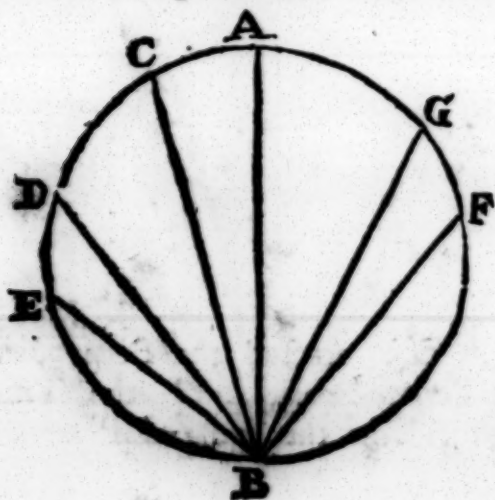


Point C to AC is erected the Perpendicular CB, meeting the Perpendicular in B; the Time of Descent through AC in the inclined Plane will be [by
Corol.

Corol. 1. Probl. 5.] equal to the Time of the Fall through AB in the Perpendicular. I say also, the Time of Descent through CB will be equal to the same Time through AB. Draw CD parallel to AB, and DB to AC, and [by 34 *El.* 1.] CD will be equal to AB: and by reason of the Angle ACB being right in the Semicircle, the Angle CBD will be a right one. Wherefore since from the Point B, upon CB, is erected at right Angles BD meeting with the Perpendicular in D; the Time of Descent through CB will be [by Corol. Probl. 5.] equal to the Time of Descent through CD: but CD is equal to AB, whence the Time of Descent through CB will be equal to the Time of Descent through AB.

THE same thing may be otherwise shewn thus. The Time of Descent through AB is to the Time through EB, in a subduplicate Ratio of AB to EB, that is, (by reason AB, BC, EB, are continual Proportionals) as AB to BC, or BC to BE; but [by Theor. 36.] the Time of Descent through BC is to the Time through EB in the same Ratio of BC to EB: wherefore since the Times of Descent through AB and BC have the same Ratio to the Time of Descent through EB, they will be equal. *Q. E. D.*

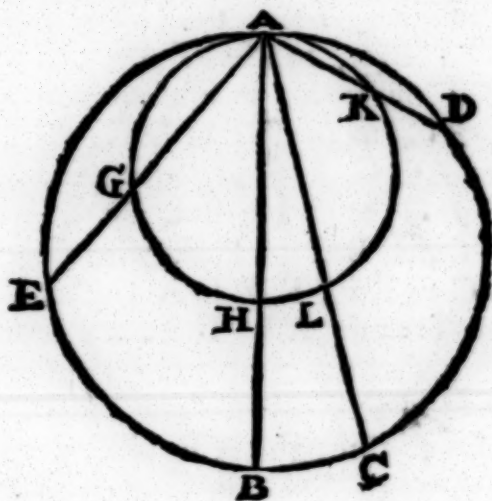
Cor. 1. IF the Perpendicular AB is drawn, and



upon

upon the Diameter AB is described a Circle, all the Planes drawn from the Point B , or from the Point A to the Circumference of the Circle, will be run over in the same Time: namely, the Planes AB , CB , DB , EB , FB , GB , will be passed over in the same Time.

Cor. 2. If in the highest Point A , any number of Circles, ABD , AGK , touch each other; and any number of Planes proceed from A , as AB , AC , AD ,

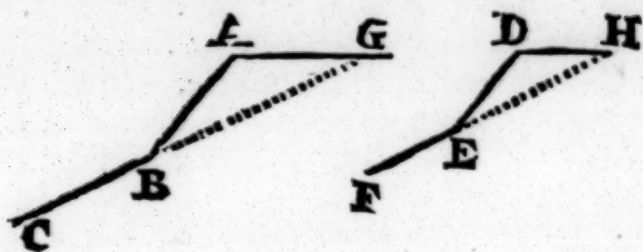


AE , intersecting the Circles: the Parts GE , HB , LC , KD , will be passed over in an equal Time, if the Motion is begun at the highest Point A .

THEOR. XL.

If two heavy Bodies descend in two or more Planes, inclined alike, and proportional; the Times spent in running through them, will be in a subduplicate Ratio of the Lengths of the Planes.

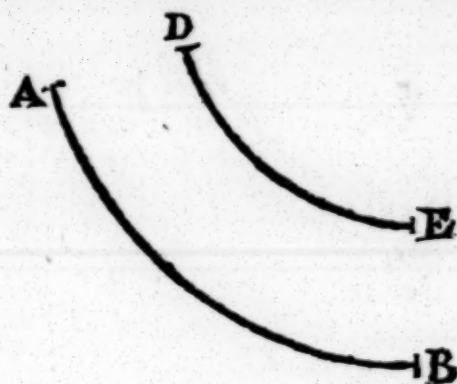
LET any heavy Body run through the Planes AB , BC , but another heavy Body run through the Planes DE , EF , inclined alike to the Horizon, and proportional; that is, that the Angles BAG , EDH , are



are equal, as are also BGA , EHD ; and AB is to BC as DE to EF . I say, the Time wherein AB , BC , are passed over, to the Time wherein DE , EF are passed over, has a subduplicate Ratio of the Planes AB , BC , to the Planes DE , EF . By reason the Triangles ABG , DEH , are equi-angular, AB is to DE as BG to EH ; but, by Hypothesis, as AB to DE so is BC to EF , wherefore as BG to EH so is BC to EF ; and so is [by 12 *El.* 5.] GC to HF . But because AB , DE , are inclined alike, they will be passed over just after the same manner, as if they were Parts of the same Plane: so that the Time of Descent through AB will be to the Time of Descent through DE , in a subduplicate Ratio of AB to DE ; and the Time of Descent through GC is to the Time of Descent through HF , in a subduplicate Ratio of GC to HF , or in a subduplicate Ratio of AB to DE . But the Time of Descent through GB is to the Time of Descent through HE , in a subduplicate Ratio of GB to HE , or AB to DE ; so that [by 19 *El.* 5.] the Time of Descent through BC , after the Descent from G or A , is to the Time of Descent through EF after the Descent from H or D , in a subduplicate Ratio of AB to DE ; that is, as the Time of Descent through AB to the Time of Descent through DE . So that [by 12 *El.* 5.] the Time of Descent through AB , BC , will be to the Time of Descent through DE , EF ,
as

as the Time of Descent through AB, to the Time of Descent through DE, or in a subduplicate Ratio of AB to DE: but by reason AB is to DE as BC to EF, AB will be to DE as AB, BC to DE, EF. So that the Time of Descent through AB, BC, will be to the Time of Descent through DE, EF, in a subduplicate Ratio of AB, BC, to DE, EF. *Q. E. D.* The same thing, in like manner, might be shewn, if there were more Planes on both Parts inclined and proportional; whence the Proposition is manifest.

Cor. If there are two curve Superficies AB, DE, similar and similarly posited, these very little differ from an infinite Number of Planes, infinitely small,



and proportional, and inclined alike to one another; so that the Time of Descent through the Superficies AB, will be to the Time of Descent through the Superficies DE in a subduplicate Ratio of AB to DE.

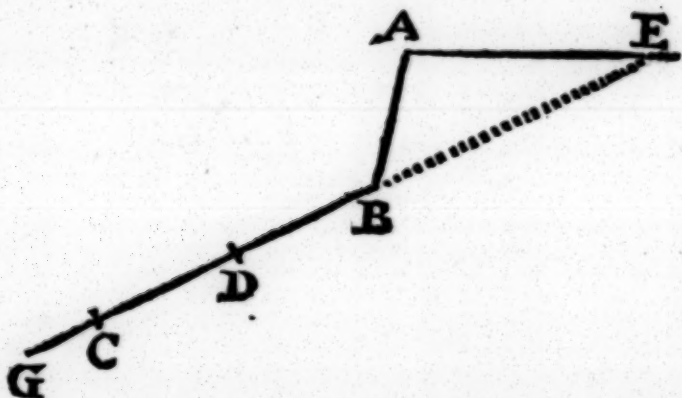
PROBL. VI.

THE Space AB being given in any inclined Plane, and which is passed over in a given Time by a heavy Body falling from rest; to find a Space in another contiguous Plane BG, that will be passed over in an equal Time, by the heavy Body continuing its Motion along this second Plane.

H O O O

THROUGH

THROUGH A draw the Horizontal right Line A E, and produce B G to E; make B D equal to A B, and take E C a third proportional to the right Lines E B, E D. Then B C will be the Space in the



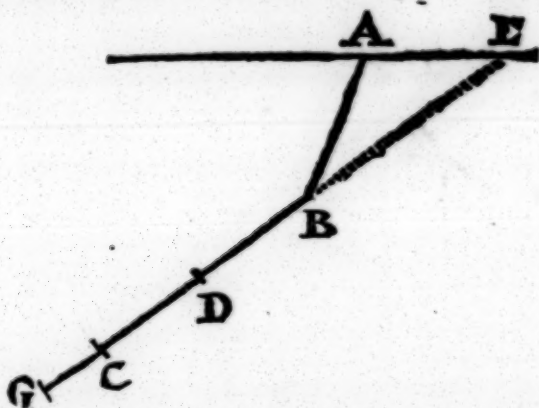
second Plane that is passed over by the heavy Body continuing its Motion, in a Time equal to that, wherein it passed over A B in the first Plane. For let A B or B D express the Time of Descent through A B, whence [by Corol. Theor. 36.] E B will express the Time of Descent through E B. But the Time of Descent through E B is to the Time of Descent through E C, in a subduplicate Ratio of E B to E C, that is, as E B to E D; but E B is the Space that is passed over in the Time as E B; so that E C will be the Space that is passed over in the Time as E D, and consequently B C is the Space that is passed over in the Time as D B or A B, after the Fall from E or A. Which was to be found out.

PROBL. VII.

THE Space A B being given in an inclined Plane, which is passed over in a given Time by a heavy Body falling from rest; as also the Space B C in another contiguous Plane, wherein the heavy Body continues its Motion: to find the Time wherein that given Space B C will be passed over.

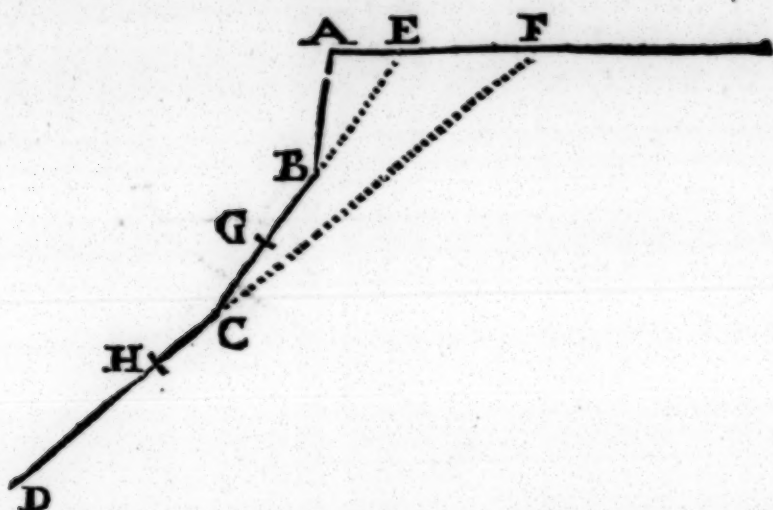
THROUGH

THROUGH A draw the Horizontal Line AE, which let BC meet produced in E: betwixt EB, EC, let ED be a mean Proportional. And if AB represents the Time wherein AB is passed over,



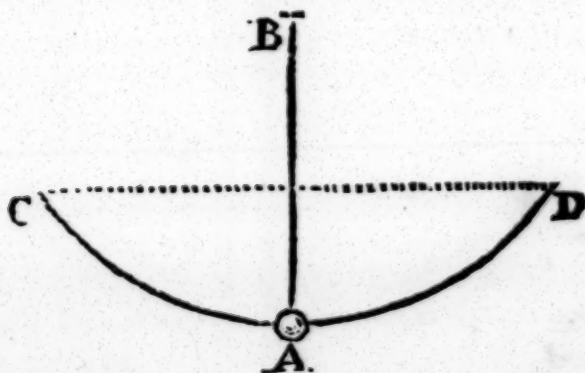
BD will represent the Time sought, wherein BC will be passed over. For the Time of Descent through AB is to the Time of Descent through EB, as AB to EB; so that EB will express the Time wherein the heavy Body will fall through EB: but the Time of Descent through EB is to the Time of Descent through EC, in a subduplicate Ratio of EB to EC; or, by reason EB, ED, EC, are continual Proportionals, as EB to ED: but EB is as the Time of Descent through EB, whence DB will be as the Time of Descent through BC. And consequently the Time of Descent through AB, will be to the Time of Descent through BC, as AB to BD. *Q. E. I.*

Cor. HENCE if a heavy Body is carried successively through many inclined Planes AB, BC, CD, the Time may be assigned, wherein it will be moving through each of them: for produce BC, CD, that they may meet with the Horizontal right Line drawn through A, in the Points E and F, and let EG be a mean Proportional betwixt EB and EC; as FH betwixt



betwixt FC and FD: And if AB expresses the Time of Descent through AB, BG will express the Time of Descent through BC, and CH will express the Time of Descent through CD.

Def. If any heavy Body A, is hung by a fine String moveable about a Center B; the heavy Body, together with the String, is called a Pendulum. Now if the Pendulum is swung about B,



so that the heavy Body may describe the Arch CAD, the same Motion will happen to this heavy Body, as would to any Body descending by its Gravity

vity along the spherical Superficies CAD , if that Superficies was perfectly hard and smooth. For we suppose the Motion about the Point B to be intirely free, and we abstract from the Consideration of the Air's Resistance, which in heavy Pendulums is very inconsiderable: if the Pendulum is carried to the Situation BC , and let fall from thence, the heavy Body in descending will describe the Arch CA , and in the Point A it will have that Velocity which is acquired by falling through CA : with which Velocity it will endeavour to go out by the Tangent in the Point A , [by the first Law.] But since it is by the String AB kept in the Circumference CAD , it will ascend through the Arch AD to the same Height, namely, to D , with that from whence it fell, [by Corol. Theor. 37.] where having lost all its Velocity, it will again begin to descend by its own Gravity; and in the Point A it will acquire the former Velocity, wherewith it will ascend to C : and so by ascending and descending, it will perform continual Vibrations in the Circumference CAD . And if the Motions of Pendulums met with no Resistance from the Air, and if there was no Friction at the Center of Rotation B , the Vibrations of Pendulums would never cease; but from these Causes, the Velocity of the Pendulum in the Point A is a little, tho' very insensibly diminished in every Vibration; whence it falls out, that the Ball of the Pendulum does not return precisely to the same Point, but the Arches which it describes constantly become shorter, till at length they grow insensible.

THEOR. XLI.

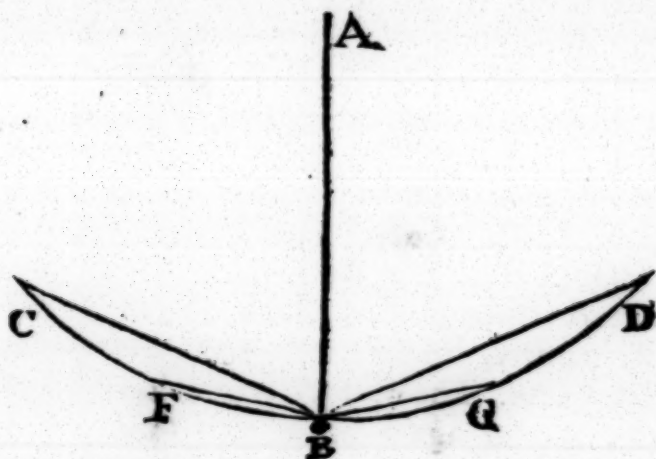
THE small Vibrations of the same Pendulum, tho' they should be unequal, are almost and to Sense performed in the same Time.

LET AB be a Pendulum, which by Vibrating describes the unequal Arches CBD , FBG ; I say, the

Q

Times

Times taken up in describing them are nearly equal : or the Vibration in the Arch CBD will be nearly performed in a Time equal to that, wherein the Vibration in the Arch FBG will be performed, if the



Arches CB, FB, are not too large. Draw the Subtenses CB, FB, DB, GB, and because the Arches are supposed very small, they differ not much either in Length or Declivity from their Subtenses or Chords: and consequently the heavy Body takes up equal Times, whether it is carried along the Arches CB, FB, or the Subtenses of those Arches; but the Times of Descent through the Subtenses of Arches are equal, [by Theor. 37.] wherefore the Times of Descent through the Arches CB, FB, will be nearly equal; and therefore the Doubles of these Times, namely, wherein the unequal Arches CBD, FBG, are described by vibrating, will be also equal. Wherefore the Vibrations of the same Pendulum, though running out into unequal Arches, are to Sense at least of equal Duration. *Q. E. D.*

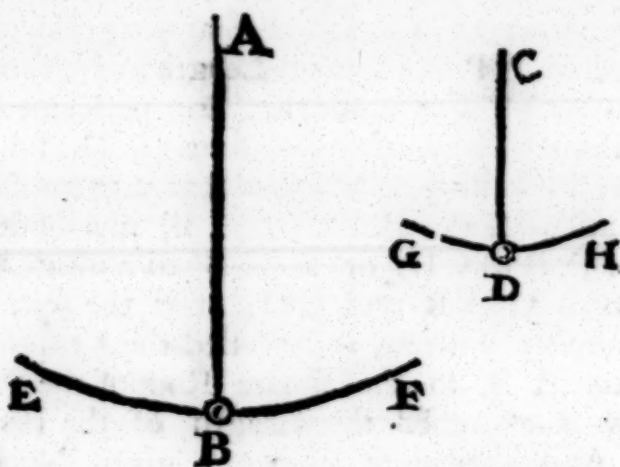
EXPERIENCE agrees with this Theorem; for two Pendulums of equal Lengths being put in motion, and whereof one describes much greater Arches than

than the other, they will nearly observe equal Times in their Vibrations; so that in a hundred Vibrations there will scarce be the Difference in Time of one Vibration.

THEOR. XLII.

THE Durations of the Vibrations of two Pendulums, describing similar Arches, are in a subduplicate Ratio of the Lengths of the Pendulums.

LET there be two Pendulums AB, CD, vibrating in similar Arches EBF, GDH; the Time of a Vibration of the Pendulum AB will be to the



Time of a Vibration of the Pendulum CD, in a subduplicate Ratio of the Length AB to the Length CD. For because the Arches EB, GD, are similar, and posited alike, the Time of Descent through EB will be [by Theor. 39.] to the Time of Descent through GD, in a subduplicate Ratio of EB to GD; but the Time of Descent through EB is half a whole Vibration in the Arch EBF, as the Time of Descent through GD is half a Vibration in the Arch GDH; so that the Time of Vibration of the

Pendulum in the Arch $E B F$ will be to the Time of Vibration of the Pendulum in the Arch $G D H$, in a subduplicate Ratio of $E B$ to $G D$; that is, by reason the Arches $E B$, $G D$, are similar, in a subduplicate Ratio of the Semidiameter $A B$ to the Semidiameter $C D$, or in a subduplicate Ratio of the Length of the Pendulum $A B$ to the Length of the Pendulum $C D$. *Q. E. D.*

Cor. THE Lengths of Pendulums are in a duplicatae Ratio of the Times wherein the Vibrations are performed.

SINCE the Durations of the Vibrations are reciprocally as the Number of Vibrations performed in the same Time, there may be easily found from a given Number of Vibrations that are performed by a Pendulum $A B$ of a known Length in a given Time, the Number of Vibrations that are performed in the same Time by any other Pendulum $C D$ of a known Length; by taking a Number that is to the Number of Vibrations of the Pendulum $A B$, in a subduplicate Ratio of $A B$ to $C D$, or as $A B$ to a mean Proportional betwixt $A B$ and $C D$; or as the square Root of a Number whereby is expressed the Length of the Pendulum $A B$, to the square Root of the Number whereby is expressed the Length of the Pendulum $C D$. And alternately from the given Number of Vibrations that are performed in the same Time by two Pendulums $A B$, $C D$, and the Length of one, namely, $A B$ being given; there will be given the Length of the other $C D$, viz. by making the Square of the Number of Vibrations of the Pendulum $C D$ to the Square of the Number of Vibrations of the Pendulum $A B$, so the Length of $A B$ to $C D$ the Length sought.

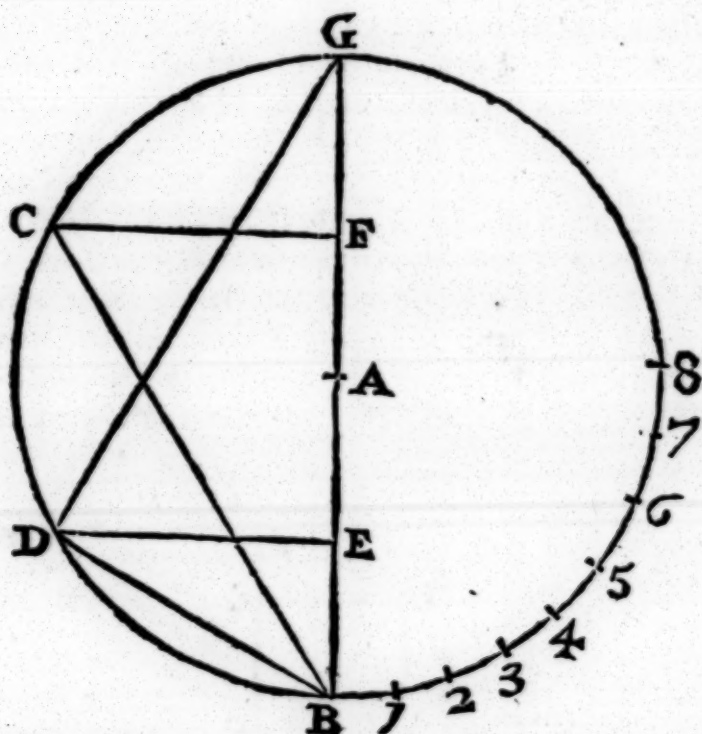
THEOR. XLIII.

THE Velocity of a Pendulum, in its lowest Point, is as the Subtense of the Arch which it has described in its Descent.

LET

Lect. 15. *Natural Philosophy.* 229

LET AB be a Pendulum, which by its Motion describes the Circle BDCG; I say, the Velocity acquired by falling from D to B, is to the Velocity acquired in B by falling from C to B, as the Chord of the Arch BD, to the Chord of the Arch BC. Through the Points D, C, draw the Horizontal right



Lines DE, CF; and the Velocity the heavy Body acquires by descending through EB will be to the Velocity it will acquire in its Descent through GB, in a subduplicate Ratio of EB to GB, that is, by reason EB, DB, GB, are continual Proportionals, as DB to GB. By the same reason, the Velocity acquired by the moveable Body, in falling through GB, is to the Velocity acquired in the Fall through FB, as GB to CB. Wherefore by Equality, the Velocity acquired, in the Descent of the heavy Body through EB, will be to the Velocity acquired

in the Descent through FB , as DB to CB ; but the Velocity acquired in the Descent through the Arch DB , is the same with the Velocity acquired in the Perpendicular through EB ; and the Velocity acquired in the Descent through the Arch CB , is the same with the Velocity acquired by the Descent in the Perpendicular through FB . Wherefore the Velocity acquired in the Descent through the Arch DB , will be to the Velocity acquired in the Descent through the Arch CB , as the Subtense DB to the Subtense CB . *Q. E. D.*

Cor. 1. LET GB be a Perpendicular of any Length, and let the Velocity acquired by the Descent of a heavy Body from G to B , be expressed by GB ; upon which, as a Diameter, let be described a Semi-circle $GDCB$, and from any Point of the Diameter E let be erected the Normal ED , meeting the Circumference in D , and let be drawn the Chord GD ;



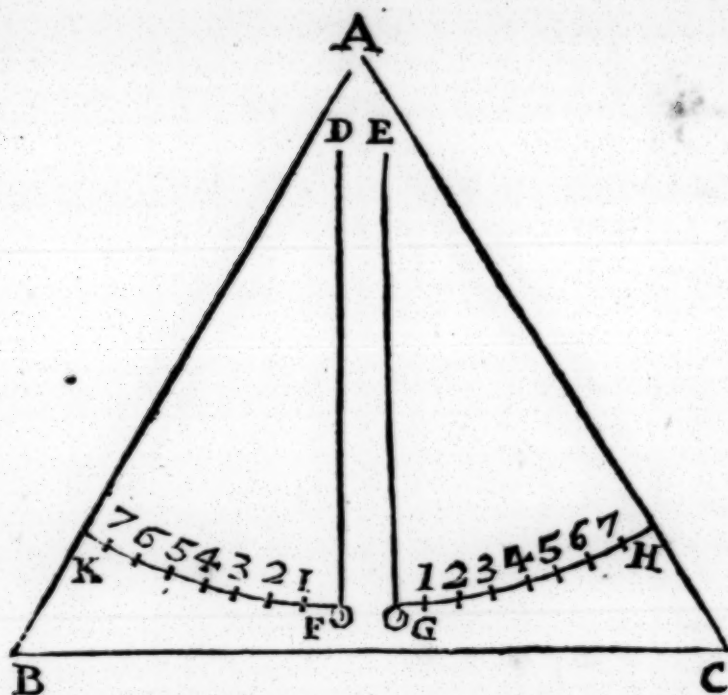
this will be as the Velocity acquired by the heavy Body in falling from the Height GE : for by reason BG , GD , GE , are continual Proportionals, the Ratio of BG to GD , will be subduplicate of the Ratio of BG to GE , so that BG will be to GD , as the Velocity acquired by falling from the Height GB , to the Velocity acquired by falling through GE . In like manner, the Velocity acquired by falling

ling through GB , is to the Velocity acquired by the Fall through GF , as GB to GC : so that the Velocities acquired by heavy Bodies, in falling through the Heights GE , GF , are as the Chords GD , GC .

Cor. 2. If the Arches B_1, B_2, B_3 , &c. are taken in such a manner, that their Subtenses are as 1, 2, 3, &c. respectively; and a certain Force acting on it, should compel the Pendulum to move upwards through the Arch B_1 , but another Force make it move through the Arch B_2 , and another through the Arch B_3 , the Velocities of the Pendulum put in motion by these Forces, will be in the point B , respectively as 1, 2, 3.

By the help of this Theorem, various Velocities in any given Ratio may be communicated to a moveable Body; and other Velocities acquired by the Percussion of another Body, may be compared amongst themselves, and with others that were given at the beginning.

Let ABC be a Triangle made of Wood, wherein near the Angle A let be taken two Points D, E , whose Distance let be such, that the two Pendulums DF, EG , hanging freely from thence, may touch each other; and with the Centers D, E , and Interval DE , or EG , let be described the Arches of Circles FK, GH , in which let be taken the Portions $F_1, G_1, F_2, G_2, F_3, G_3, F_4, G_4$, &c. in such a manner, that their Subtenses may be as 1, 2, 3, 4, &c. respectively; and if the heavy Body F be raised up to the Point 5 in the Arch KF , but G to the Point 3 in the Arch GH , and are both let down at once, [by Theor. 41.] they will arrive in the same Time at the lowest Points, and the Velocities wherewith they will strike one another, will be as 5 and 3; but if after the Stroke the moveable Body G ascends to 5 in the Arch GH , and the moveable Body F ascend to 3 in the Arch FK , the Velocities of the move-



able Bodies F and G will be respectively as 3 and 5, and towards the contrary Parts. After this manner it will be easy to try by Experiments the Rules of Motion as well in hard as elastick Bodies, which we have demonstrated in the 13th and 14th Lectures.

SINCE the very small Vibrations of these Pendulums are nearly of equal Duration, altho' the Arches they describe are unequal; hence the great Monf. *Huygens* has shewn the vast Use of Pendulums to regulate the Motions of Clocks. For although *Gallileo*, the Author of this Science, first made use of Pendulums in Astronomical and Philosophical Observations, which require an accurate Measure of Time; yet *Huygens* first adapted Pendulums to Clocks, and proved by Experience that such Clocks were far more correct than the former, which were regulated by Horizontal Ballances. From that time these Pendulum
Clocks

Lect. 15. *Natural Philosophy.* - 233

Clocks have been in common use, whereof some have been so elaborately made, that they give the Measure of Time much more correct than the Sun, which shews only the apparent or relative Time, but not the true and absolute: whence it happens, that Pendulum Clocks shew at Times the Hour different from the apparent one, and sometimes exceeding the Time shewn by a Sun-Dial by fifteen or sixteen Minutes, and sometimes falling as much short of it in Time: nor does the Sun and Pendulum Clock shew the same Point of Time, except four times a year.

ALTHOUGH the Vibrations of the same Pendulum (notwithstanding it describes unequal Arches) are nearly and to Sense of equal Duration; yet since they are not altogether and geometrically such, but the greater are a somewhat longer time in describing than the less, and the Vibrations differ from each other a small Quantity in Time; from many of these very small Differences at length there do arise a sufficient large Difference, and this is found to be so by Experience: For if, as it sometimes happens in cold Weather, the Wheels are clogged, so that they communicate a less Force to the Pendulum, the Vibrations become less, and the Time shewn by the Clock is too fast; if the Wheels are too glib, and force the Pendulum to describe larger Arches, then the Times are pointed out too slowly. And from some late Experiments mentioned in the *Philosophical Transactions*, it appears that the Pendulum of a Clock placed in *vacuo*, the Resistance of the Air being thus removed, performed its Vibrations in larger Arches, and took up more Time in each Vibration. Wherefore that the Vibrations of a Pendulum might be reduced to an exact Equality, and that its Reciprocations, whether performed in greater or less Arches, might be made in equal Times; Monsr. *Huygens* found out a Method, whereby the Ball of a Pendulum may be

be always carried along the Arch of a Cycloid. We shall now demonstrate in the following Theorems, that the Times of Descent through any Arches of the same Cycloid to its lowest Point, which is supposed to be its Vertex, are equal to one another; so that if the Ball of a Pendulum is always moved in the Arch of a Cycloid, the Times of its Vibrations will be exactly equal amongst themselves, whether the Pendulum describes greater or less Arches.

T H E O R. XLIV.

IF with the Center C, and any Interval CA, there is described the Quadrant of a Circle AHB, and in the right Line AC a moveable Body descends with this Law, that its Velocity in any Point P shall be always as PL, the Sine of the Arch AL; the Time wherein the Body descends from A to C, will be equal to the Time wherein it might run through the Circumference AHB, with a uniform Velocity as CB, which is acquired at the last by the falling Body. Besides, the Time of the Fall through any Space AF, will be to the Time of the Fall through the Space Ap, as the Arch AH to the Arch Al; and the Force wherewith the Body is accelerated in any Place F, will be as FC, the Distance of that Place from the Center.

LET the Circumference AB be distinguished into innumerable infinite small Parts L, L, L, L, and let be drawn FH, PL, Pl, perpendicular to AC; join HC, and let HK be perpendicular to PL. Because the Triangles FHC, KHL, are equi-angular, (for besides the Angles at F and K being right ones, the Angle FHC is equal to the Angle KHL, the Angle KHC being the Complement of both to a right Angle) FH will be to HC as KH or FP to HL: but [by Hypothesis] FH is as the Velocity of the moveable Body in the Point F, namely, where-
with

BESIDES the Time wherein the moveable Body descends from A to F, is equal to the Time wherein the Arch AH is passed over; and the Time wherein it descends from A to p , is equal to the Time wherein the Arch Al is described; but the Time wherein the Arch AH is passed over, is to the Time wherein the Arch Al is passed over, (since they are both described with the same Velocity) as the Arch AH to the Arch Al; wherefore the Time of Descent from A to F, will be to the Time of Descent from A to p , as the Arch AH to the Arch Al; and consequently by Division the Time of Descent through Fp will be as the Arch Hb. *Q. E. D.*

MAKE the Arches HL, bl, equal; whence the Time of Descent through FP will be equal to the Time through fp: and by reason the Triangles KHL, FHC, as also kbl, fbc, are similar; KL will be to HL or bl, as FC to CH or Cb: Also bl is to kl as Cb to Cf; and consequently, by Equality, KL will be to kl as CF to Cf: but KL is as the Increment of the Velocity acquired whilst the Body passed over FP, and kl is as the Increment of the Body's Velocity, whilst it passed over in an equal Time the infinite small Line fp; but the Forces wherewith the Body is accelerated in the Places F and f, are as the Increments of the Velocities generated in equal Times; the accelerating Forces therefore of the moving Body in the Places F and f will be as the right Lines KL, kl; that is, the force wherewith a Body is urged in F, is to the force wherewith a Body is urged in f, as the Distance GF to the Distance CP. The accelerating Forces therefore are in any Places as their Distances from the Center. *Q. E. D.*

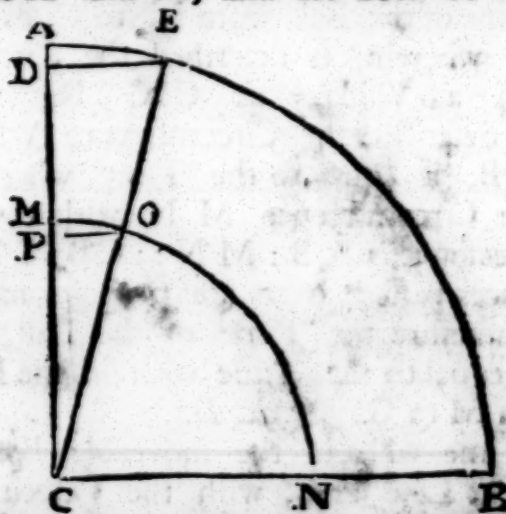
Cor. Hence conversly if a moveable Body in descending from A to C is urged by a Force which is as its Distance from the Center; and that Force at the

the beginning of Motion is expressed by the right Line DE, the Arch AE being taking infinitely small, the Velocities of the same Body in any Places F, f, will be expressed by the Sines FH, fh, and the Times by the Arches AH, Ab, and the Increments of the Velocities; or if the Arches increase equally, the accelerating Forces will be expressed by the Increments of the Sines.

THEOR. XLV.

If a moveable Body be sollicitated in the right Line AC towards the Point C, with Forces that are proportionable to the Distances from the Point C, from whatever Height it falls, it will arrive at the Point C always in the same Time; and that Time is to the Time wherein a moveable Body may pass over the same way, with a Velocity that is uniform and equal to that which is acquired at last by falling, as half the Circumference of a Circle to its Diameter.

LET two moveable Bodies be let fall at once from the Points A and M, and let both be sollicitated by



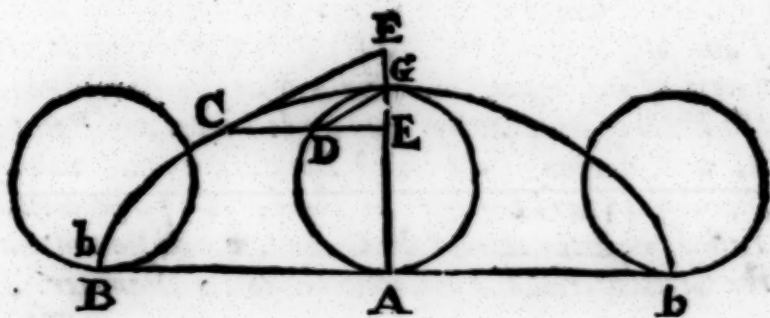
Forces that are proportionable to their Distances from the Point C; I say, both the Bodies will arrive

at the point C in the same Time. With the Center C, and Intervals CA, CM, let be described two Quadrants of Circles AB, MN, and let the Force whereby the Body A is urged, or, which is the same thing, its Velocity at the beginning of Motion be represented by DE, the Sine of the infinitely small Arch AE; it is manifest from the Corol. of the preceding Theor. that its Velocity, after the Fall to C, will be represented by the right Line CB. But, by Hypothesis, the Force wherewith the moveable Body is accelerated in A, is to the Force wherewith the moveable Body is accelerated in M, as CA to CM, or as DE to PO, by reason the Arches AE, MO, are similar: wherefore if DE expresses the Velocity of the moveable Body at the beginning of the Fall from A, PO will express the Velocity of the moveable Body at the beginning of the Fall from M; and consequently [by the same Corol.] CN will express the Velocity of the Body in C after its Fall through MC. Besides, the Time of the Fall from A to C, is equal to the Time wherein may be described the Circumference AB, with a uniform Velocity as CB, and the Time of the Fall from M to C, is equal to the Time wherein is described the Circumference MN with a Velocity as CN. But the Time wherein is described the Circumference AB with the Velocity CB, is equal to the Time wherein is described the Circumference MN with the Velocity CN, (by reason it is $AB : MN :: CB : CN$, namely, the Spaces passed over are proportionable to the Times) wherefore the Time of the Fall from A to C, will be equal to the Time wherein the Body descends from M to C. *Q. E. D.*

THE Time wherein the moveable Body passes over the right Line AC, with the Velocity CB, is to the Time wherein it runs through the Arch AB with the same Velocity, as the right Line AC to the

the Arch AB , or as its Double to the Double of this; that is, as the Diameter of the Circle to half its Circumference: but the Time of Descent through the Arch AB is equal to the Time of Descent to C ; whence the Time wherein a Body is carried through the right Line AC with a Velocity as CB , will be to the Time of the Fall to C , as the Diameter of the Circle to half the Circumference. *Q. E. D.*

Defin. If a Circle insisting on the right Line Bb , and touching it in its Point b , is supposed to revolve upon that right Line, and by its Circumference be-



ing continually applied to it, to measure a right Line BAb equal to that Circumference, till the Point b being carried aloft, and so describing in its Motion the Curve BGb , and having finished its Circuit, it again touches the right Line BAb in the Point b ; the Curve described by the Motion of the Point is called a Cycloid. And the Figure $BGDAB$ is called the Figure of the Cycloid; the right Line GA bisecting the Base perpendicularly, the Axis of the Cycloid; and the Point G the Vertex of the Cycloid. The Circle is named the generating Circle, and its Point b the describing Point.

LEMMA.

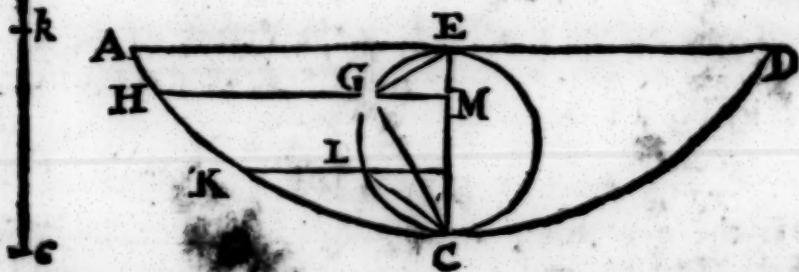
If the generating Circle is placed about the Axis of the Cycloid, and from any Point of the Cycloid C

is drawn ordinately to the Axis the right Line CE, meeting with the Circumference of the Circle in D; the right Line CD will be equal to the circular Arch GD, the Arch of the Cycloid GC will be equal to double the Chord GD; and the Semi-cycloid BCG will be equal to twice the Diameter AG: but the right Line CF touching the Cycloid in C, will be parallel to the Chord DG. All these things have been demonstrated by Dr. Wallis, and others who have wrote concerning the Cycloid.

THEOR. XLVI.

IN a Cycloid whose Axis is placed perpendicularly to the Plane of the Horizon, and its Vertex downwards, the Times of the Descents wherein a moveable Body urged by the Force of its Gravity, and let fall from any Point in it, will arrive at the lowest Point, are equal amongst themselves; and have to the Time of the perpendicular Fall through the Axis of the Cycloid, the Ratio that half the Circumference of a Circle has to its Diameter.

LET ACD be a Cycloid, whose Axis is CE, generating Circle ECG. Since the right Line touching the Cycloid in any Point H, is parallel to the Chord CG drawn in the generating Circle placed on the Axis of the Cycloid; it is manifest the moveable Body in its Descent will be accelerated with the same Force in the Point H, as if it descended in the right Line GC; but the Force wherewith it is accelerated in GC, is to



the Force of Gravity as MC to GC : but as MC to GC , so is GC to CE , [by Cor. 8. Prop. El. 6.] wherefore the Force wherewith the moveable Body is accelerated in the Point H , is to the Force of Gravity as GC to CE . By the same reason, the Force of Gravity is to the Force wherewith the moveable Body is accelerated in any other Point K , as CE to CL ; wherefore by Equality the Force wherewith the moveable Body is accelerated in H , is to the Force wherewith it is accelerated in K , as GC to LC , or as twice GC to twice LC , that is, as the Curve of the Cycloid HC to the Curve KC . The Forces therefore wherewith the moveable Body in descending along the Cycloid is accelerated, are as the Lengths of the Curve run through. Let us now put the right Line ac equal to the Length of the Curve AC , and let any moveable Body be supposed to be urged with the same Forces in the right Line ac towards c , as the moveable Body is urged, in descending along the Curve AC ; but the Forces wherewith the moveable Body is urged, in any Points H and K of the Cycloid, are as the Lengths HC , KC , or bc , kc : that is, the Forces in any Places are as the Distances of those Places from the Point c , and consequently [by the preceding Theor.] the Times of Descent from any Height will be equal. Because therefore in the corresponding Points of the Cycloid and right Line ac , the accelerating Forces are equal, the Increments of the Velocities will be also equal: as, for example, putting $AH = ab$, the Accelerations in the Points H and b will be equal, as also in the Points K and k , so that AK is made $= ak$; and in like manner, in all the other Points of both Lines, which mutually answer to one another, the Increments of the Velocities will be equal: so that if the Bodies begin to descend from corresponding Points, the Sums of the Increments, or the Ve-

R

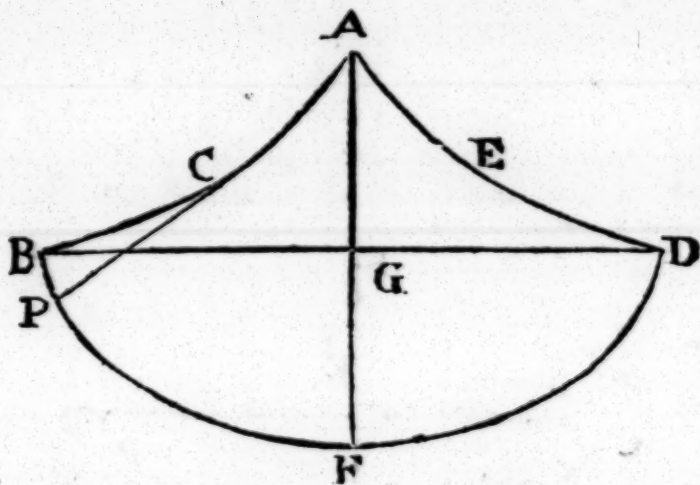
locities

locities acquired in describing equal Spaces, will be equal; and consequently the Times wherein these equal Spaces are described with equal Velocities, will be equal. Therefore the Time of Descent from a to c in the right Line ac , is equal to the Time of Descent from A to C in the Cycloid; and the Time of Descent from b to c in the right Line bc , is equal to the Time of Descent from H to C in the Cycloid; and in like manner, the Time of Descent through $K C$, is equal to the Time through $k c$, if the beginning of the Fall is from the Points k, K : and so of the rest. But the Time of the Fall from a to c , is equal to the Time of the Fall from b to c , or from k to c ; wherefore the Time of Descent in the Cycloid from A to C , will be equal to the Time of Descent from H to C , or from K to C . The Times therefore of Descent, wherein a moveable Body, let fall from any Point in the Cycloid will arrive at the lowest Point, are equal amongst themselves. *Q. E. D.*

MOREOVER, the Time of the Fall from a to c , is to the Time wherein ac , or $2 EC$, is passed over with the Velocity acquired at the last, as the Circumference of a Circle to its Diameter; but the Time wherein $2 EC$ is passed over with the same Velocity, is equal to the Time wherein the moveable Body, falling by its Gravity, descends through EC , the Axis of the Cycloid: whence the Time of Descent through ac or AC , will be to the Time wherein the heavy Body descends through the Axis of the Cycloid, as the Semi-Circumference of a Circle to its Diameter.

Cor. THE Time wherein a heavy Body descends in the Cycloid through the Arch AC , and ascends through CD , that is, the Time of the Motion in a Cycloid ACD , is to the Time of the perpendicular Fall through the Axis of the Cycloid, as the whole Circumference of a Circle to its Diameter.

HENCE if the Ball of a Pendulum performs its Vibrations in a Cycloid, whether it describes greater or less Arches, each Oscillation will be always finished in equal Times. Now *M. Huygens*, in the third Part of his *Horologium Oscillatorium*, hath delivered a Method whereby the Ball of a Pendulum may vibrate in a Cycloid or any other Curve: namely, by finding out a Curve, by whose Evolution a given Curve will be described; then two Plates must be bent into the same Curvature, betwixt which the Ball being suspended by a String of a determined Length, will describe, not a Circle, but some other Curve. Let there be two Plates *ACB*, *AED*, curved into similar and equal Figures; and from the Point *A* let be

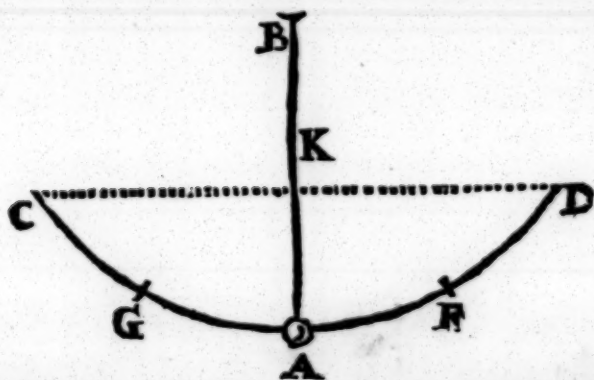


suspended the Thred of the Pendulum, which whilst the Pendulum oscillates, may be applied to the Plates *ACB*, *AED*, which it constantly touches: by the Application of the Thred to the Plates, the Motion of the Pendulum in a Circle is continually prevented, and its Ball is carried thro' the Curve *BPF D*. The Curve *ACB* or *AED* is called the Evolute, and the Curve *BPF D* is said to be described by Evolution. Now if the Curves *ACB* or *AED* are two Semi-

Cycloids, whose Axes or Diameters of the generating Circles are equal to FG or AG , namely, to half the Length of the Pendulum; the Curve $BPFD$ through which the Ball descends, becomes a whole Cycloid, whose Axis FG is half the Length of the Pendulum, as has been demonstrated by *Huygens* and others.

SINCE the Portion of the Cycloid near the Vertex F is described by the Motion of a String whose Length is AF , and a Circle whose Center is A , Semidiameter AF is described by the same String; that Circle passing through F , almost coincides with the Portion of the Cycloid near the Vertex F , and is of equal Curvity with it. A heavy Body therefore is carried to F , through a small Arch of a Circle, and through the Arch of a Cycloid, to which that Circle is of equal Curvity, in the same Time.

HENCE again the Reason is manifest, why the Times of Oscillation of a Pendulum performing small Vibrations in a Circle, are equal: For if the Arches CAD , GAF , are small, they almost coincide with



the Portion of a Cycloid near its Vertex; the Axis of the Cycloid being AK , half the Length of the Pendulum. So that the heavy Body descends through the Arches of a Circle CA or GA , almost in the same Time, as it would descend through the Arches of a Cycloid nearly coinciding with them; but it descends in equal Times through any Arches of a Cycloid;

cloid; wherefore the heavy Body also will fall in equal Times through the small circular Arches CA, GA; and consequently the whole Oscillations through the Arches CAD, GAF, are performed in equal Times.

THE Time therefore wherein a Pendulum performs a very small Oscillation in a Circle, is equal to the Time wherein an Oscillation is performed through the Arch of a Cycloid, whose Axis is half the Length of the Pendulum. But the Time wherein an Oscillation is performed in a Cycloid, is to the Time of the perpendicular Fall through the Axis of that Cycloid, that is, through half the Length of the Pendulum, as the Circumference of a Circle to its Diameter. And it hence follows, that the Time of any the smallest Oscillation is to the Time of the Fall through the Length of the Pendulum, in a constant Ratio; namely that, which the Circumference of a Circle has to its Diameter multiplied by the square Root of the Number 2.

IF in different Regions of the Earth, the same Pendulum shall perform its Oscillations in unequal Times, the Times of Descent through the Length of the Pendulum in the different Regions will be likewise unequal; and where the Oscillations proceed slower, there also a heavy Body will descend slower in the Perpendicular, and in a given Time will describe by falling a less Space. And it is certain by Experiments, that in Places situated near the Equinoctial, the Oscillations of a Pendulum of the same Length are performed in a longer Time, than in Places that are more remote from the Equinoctial: so that heavy Bodies in those Regions describe in a given Time by falling a less Space, and are accelerated in their Motion by a less Force than in our Regions, which have a greater Latitude: so that it is confirm'd by Experiments, that the Action of Gravity is less in such Places as have less Latitude, than in those that are nearer the Pole.

THIS Decrease of Gravity arises from the centrifugal Force: For since from the Rotation of the Earth about its Axis, every Body endeavours to recede from the Center of the Circle it describes, by how much the greater those Circles are, by so much the greater is the centrifugal Force of the Bodies describing these Circles; which Force therefore is always as the Sine of the Distance of the Place from the Pole, which as it is greatest under the Equinoctial, is nothing at all under the Pole itself: so that the Force of Gravity is the least under the Equinoctial, but at the Pole the greatest.

BEFORE we finish this matter, we shall here give a Solution of a famous Problem, first sought out by *Gallileo*, after proposed by *John Bernouilli* to the Geometers, namely, in the beginning of the Year 1696; and solved by the famous Geometers *Sir Isaac Newton*, *Mr. Leibnitz*, *James Bernouilli*, the *Marquiss de l'Hospital*, and others. The Problem itself was thus proposed.

GIVEN in a vertical Plane two Points A and B, to assign the Way of a moveable Body, through which it descending by its Gravity, and beginning its Motion at the Point A, it may arrive in the shortest Time possible at the other Point B.

THE above-mentioned Geometers found out, that this Line would be a Cycloid passing through the Points A, B, whose Base lay in the Horizontal Line drawn through A: to demonstrate which, we shall premise the following

LEMMA.

IF A d g B is the Line of swiftest Descent, a heavy Body will descend quicker from any Point of it d, to any other Point of it g, after the Fall from A, through the Curve itself d e g, than through any other way whatever.

FOR

call a and db or LO , applied to de , be always proportionable to the Velocity wherewith de is passed over, that is, the Velocity which is acquired by falling from A to d ; then this Curve will be the Line of swiftest Descent. Let de , eg , be taken two contiguous and infinitely small Portions of the Curve; which consequently will differ but very little from right Lines: I say, a heavy Body will descend in a leis Time through the Curve deg , after its Fall from A , than through any other Way dfg . Through f draw fq parallel to eg . And let fq be supposed to be passed over with the same Celerity as eg is passed over; and let fn be perpendicular to de , as also me , gq to fq . And by reason the Triangles fne , deb , are equi-angular, as likewise fme , gei ; de is to db as fe to ne : so that it will be $ne = \frac{db \times fe}{de}$; also by

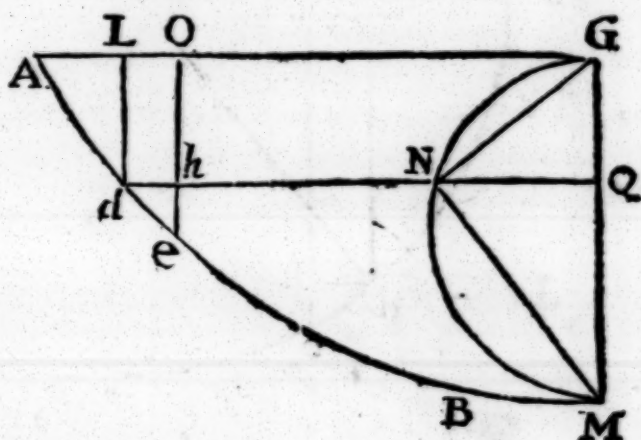
reason ge is to ei as fe to fm , it will be $fm = \frac{ei \times fe}{ge}$,

but it is $\frac{db \times fe}{de} : \frac{ei \times fe}{ge} :: \frac{db}{de} : \frac{ei}{ge} :: \frac{db \times a}{de} : \frac{ei \times a}{ge}$,

that is, ne is to fm , as the Velocity whereby ne is passed over to the Velocity whereby fm is passed over: whence ne , fm , are passed over in equal Times. And because mq is equal to eg , the Time of Descent through mq will be equal to the Time through eg ; so that the Time through fq , will be equal to the Time through neg . But by reason the Angle at q is a right one, fg is greater than fq ; so that the Time through fg , will be greater than the Time through fq , or through neg : and by reason df is greater than dn , the Time through df will be greater than the Time through dn . Whence the Time through df , fg , will be greater than the Time through dn , ng . A heavy Body therefore will descend from d to g , after its Fall from A , through the Curve

Curve *deg*, in a less Time than through any other Way; and consequently the Curve *AdegB* will be the Way of swiftest Descent.

LET ABM be a Cycloid, passing through B, whose Base let be a Horizontal right Line drawn through A; that will be the Line, on which a heavy Body descending, will proceed from A to B, in the least Time possible. Let GNM be half the generating Circle, whose Diameter GM let be called a , and let de be any infinite small Part of the Cycloidal



Curve, which will differ but insensibly from its Tangent in d ; so that it will be parallel to the right Line NM: whence the Triangles dbe , NQM , GMN , will be equiangular. Wherefore de is to db , as GM or a to GN ; and consequently $db \times a = de \times GN$; and $\frac{db \times a}{de} = GN$. But [by Cor. of Theor. 43.] GN is as the Velocity acquired by the heavy Body in falling from the Height GQ , or Ld , that is, as the Velocity wherewith the infinitely small Line de is passed over. Wherefore $\frac{db \times a}{de}$ will be proportionable to the Velocity wherewith de is passed

Curve, it will be $\frac{a^m \dot{x}}{\sqrt{\dot{x}^2 + \dot{y}^2}} = y^m$; whence $\frac{a^m \dot{x}^2}{\dot{x}^2 + \dot{y}^2} = y^{2m}$, and $a^{2m} \dot{x}^2 = y^{2m} \dot{x}^2 + y^{2m} \dot{y}^2$, and $a^{2m} \dot{x}^2 - y^{2m} \dot{x}^2 = y^{2m} \dot{y}^2$, and $\dot{x}^2 = \frac{y^{2m} \dot{y}^2}{a^{2m} - y^{2m}}$, and $\dot{x} =$

$\frac{y^m \dot{y}}{\sqrt{a^{2m} - y^{2m}}}$. Which Equation expresses universally

the Nature of a Curve, wherein a heavy Body will descend in the shortest Time, if the Velocity is as any Dignity m of the Height from whence it fell.

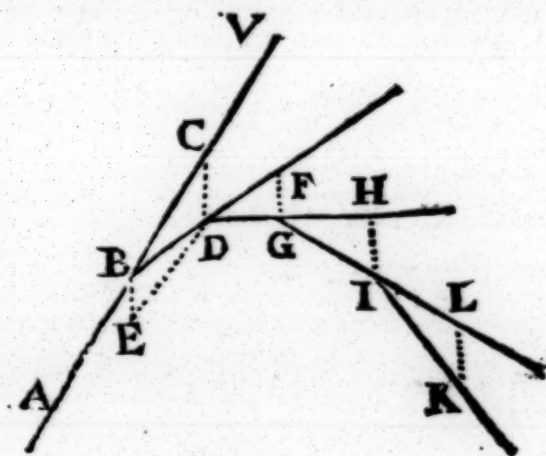


LECTURE XVI.

WE have, in the foregoing Lecture, explained the Motions of heavy Bodies on inclined Planes, or curve Superficies; and their principal Properties, as far as was consistent with our intended Brevity. It now remains that we speak of the Phenomena of Projectiles: and first of all, we ought to find out the Nature of that Line, which a moveable Body projected in free and unresisting Spaces, will describe by the Force of its Gravity. And indeed if the heavy Body is projected directly upwards or downwards, it will be moved in a right Line; and it is manifest from the former Lectures, that its Motion will be an uniformly retarded or accelerated Motion, as it is projected either upwards or downwards. But if it is projected in a horizontal Direction, or in any other that is oblique to the Horizon, it will be carried in a curve Line.

FOR

For let the Body be projected or cast from A, in the Direction A V; by the first Law of Nature, if no other Force acted on it, it would proceed always in the same right Line, with the same Velocity: so that it



would describe equal Spaces, A B, B C, in equal Times, Let us therefore distinguish the Time into equal Particles; and if after the first Particle of Time, when the moveable Body shall arrive at B, some Force is supposed to act on it, by one Impulse only; and to communicate to it a Motion, whereby it would be carried (the other Motion being taken away) in a Direction perpendicular to the Horizon, through the right Line B E, in the Time that it would describe the right Line B C; compleat the Parallelogram C B E D: it is manifest from Cor. 2. Theor. 30. that the Body, by the Motion compounded of both the others, will be moved through the Diagonal B D, and afterwards the projected Body would always proceed in this right Line, if no new Force turned it out of its Path; and it would in an equal Time describe a Space D F equal to B D. But if in the Point D, the same Force should act again with a like Impulse, whereby the Body would be

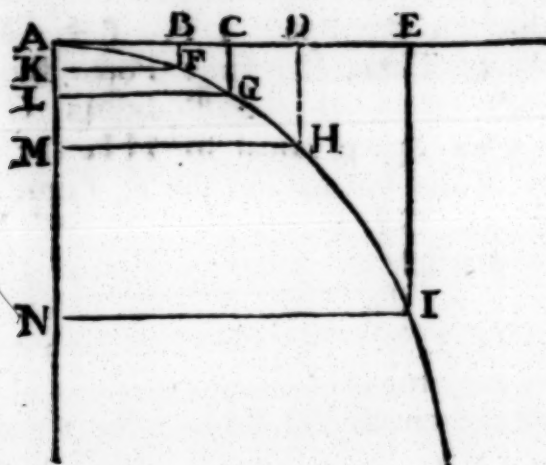
be carried downwards in that Time through a Space equal to FG: the Motion of the Body, compounded of both Motions, would be through the right Line DG, which the Body will describe in the same Time, as it would, without this new Impulse, have proceeded through the Space DF. But if after the third Particle of Time, the same Force should act again, and should impel the Body in G downwards, through a Space equal to HI; the Motion compounded of the former and this new one, will be in the right Line GI, which the Body will describe in the fourth Particle of Time: but in I the same Force acting again, the Body will be turned out of its right Path GL into the Direction IK; and after this manner the projected Body will describe by its Motion the Polygon ABDGIK. But if each Particle of Time, wherein we supposed the Force to act, is diminished *in infinitum*, and their Number is augmented: the Sides of the Polygon will be diminished *in infinitum*, and their Number will be increased *in infinitum*, and consequently the Polygon will be turned into a Curve: that is, if the Force impelling downwards is such, as, that it should act always and without ceasing, of which nature is the Force of Gravity, the projected Body, this Force acting, will be carried along a Curve.

THEOR. XLVII.

A PROJECTED Body, whose Line of Direction is parallel to the Plane of the Horizon, describes by its Motion a Parabola.

If the heavy Body is thrown by any extrinsecal Force, as that of a Gun, or the like, from the Point A, so that the Direction of its Projection is the Horizontal Line AD; I say, the Path of this heavy Body will be a Semi-Parabola. For if the Air did
not

not resist it, nor was it acted on by its Gravity, the Projectile would proceed with an equable Motion, always in the same Direction; and the Times wherein the Parts of Space A B, A C, A D, A E, were pas-



fed over, would be as the Spaces A B, A C, A D, A E, respectively. Now if the Force of Gravity is supposed to take place, and to act in the same Tenour, as if the heavy Body were not impelled by any extrinfecal Force; that Body would constantly decline from the right Line A E, and the Spaces of Descent or the Deviations from the Horizontal Line A E, will be the same as if it had fallen perpendicularly. Wherefore if the Body falling perpendicularly by the Force of its Gravity, passed over the Space A K in the Time A B, descended through A L in the Time A C, and through A M in the Time A D; the Spaces A K, A L, A M, will be as the Squares of the Times, that is, as the Squares of the right Lines A B, A C, A D, or of K F, L G, M H. But since the Impetus in the Direction parallel to the Horizon always remains the same; for the Force of Gravity, that only sollicitates the Body downwards, is not in the least contrary to it: the Body will be equally promoted

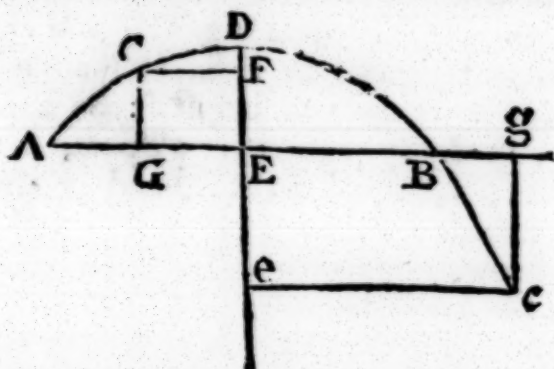
promoted forwards in the Direction parallel to the Plane of the Horizon, as if there was no Gravity at all. Wherefore since in the Time AB , the Body passes over a Space equal to AB ; but being compelled by the Force of Gravity, it declines from the right Line AB through a Space equal to AK , and BF being made equal and parallel to AK , at the end of the Time AB , the Body will be in F : So since in the Time AC , the Body passes over a Space, in the Horizontal Direction, equal to AC , and in that Time descends through a Space equal to AL ; if CG is made equal and parallel to AL at the end of that Time, the Body will be in G . In like manner, since in the Time AD , the Body is carried in the Horizontal Direction through a Space equal to AD , by the Action of Gravity, it would in the mean while descend through a Space equal to AM ; and DH being put equal to AM , at the end of the Time AD , the Body will be in H . And the Path of the Projectile will be in the Curve $AFGH$; but because the Squares of the right Lines KF , LG , MH , are proportionable to the Abscissas AK , AL , AM , the Curve $AFGH$ will be a Semi-Parabola. The Path therefore of a heavy Body projected according to the Direction AE , will be a semi-parabolical Curve. *Q. E. D.*

L E M M A.

LET ADB be such a Curve, as that the Perpendicular CG being let fall from any of its Points C to AB , the Rectangle under AG , GB , is equal to the Rectangle under CG , and a given right Line L ; that Curve will be a Parabola.

LET AB be bisected in E , and the Perpendicular DE be erected; by Hypothesis, the Rectangle under DE and L , will be equal to the Rectangle under

der AE , EB , or $AEq. =$ (by 5 *El.* 2.) $AG \times GB + GEq. = CG \times L + GEq. = EF \times L + CFq.$ Wherefore the Rectangle under DF and L will be equal to the Square of CF , which is the



Property of the Parabola. If the Point g falls in AB produced, which happens when the Curve descends below AB , the same Parabola will be the *Locus* of the Point c ; for (by 6 *El.* 2.) it is $Egq. = (ecq. =) Ag \times gB + EBq. = L \times cg + L \times DE = L \times Dc$: which is the Property of the Parabola.

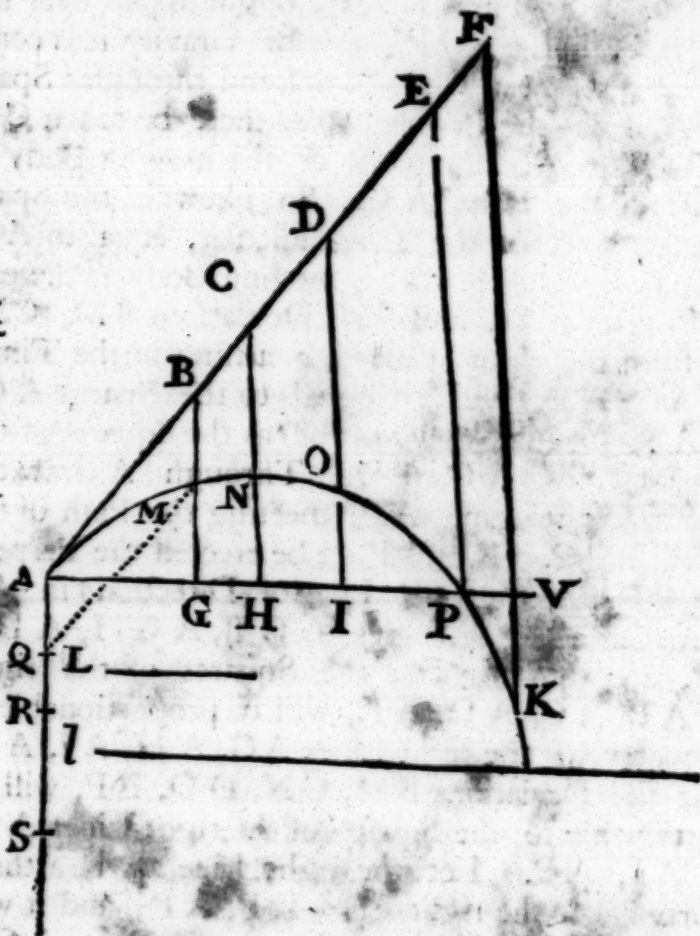
Cor. THE right Line L is the *Latus Rectum*, or *Parameter* of the Parabola.

THEOR. XLVIII.

THE curve Line, that is described by a heavy Body projected obliquely and upwards, according to any Direction, is a Parabola.

LET AF be the Direction of Projection, any ways inclined to the Horizon. Gravity being supposed not to act, the moving Body would always continue its Motion in the same right Line, by the first Law of Nature, and would describe the Spaces AB , AC , AD , proportionable to the Times. But by the Action of Gravity it is compelled continually to decline

cline from the Path AF , and to move in a Curve; I say, this Curve will be a Parabola. Let us suppose the heavy Body falling perpendicularly in the Time AB , through the Space AQ , but in the Time AC ,



through the Space AR ; and in the Time AD , thro' the Space AS ; the Spaces AQ , AR , AS , will be as the Squares of the Times, or as the Squares of the right Lines AB , AC , AD . But because the moving Body not being acted upon by Gravity, in the Time AB would pass over the Space AB , and by the Action of Gravity would in the mean while descend

descend through a Space equal to AQ , it is manifest, if in the Perpendicular BG , there is taken $BM=AQ$, the Place of the heavy Body, at the end of the Time AB , would be M . In like manner, since the moving Body, by the Impetus first impressed, in a Time as AC , ought to pass over the Space AC , but by the Force of Gravity it is compelled in the mean while to descend through a Space $=AR$; if in the Perpendicular there be taken $CN=AR$, N will be the place of the moving Body at the end of the Time AC . So likewise the Space DO being put, in the Perpendicular, equal to AS , O will be the Place of the moving Body at the end of the Time AD ; and the Deviations BM , CN , DO , from the right Line AF arising in the Times AB , AC , AD , will be equal to the Spaces AQ , AR , AS , so that they will be as the Squares of the right Lines AB , AC , AD . Through A draw the horizontal right Line AP , meeting the Path of the Projectile in P . From P let be erected the Perpendicular PE , meeting the Line of Direction in E ; and by reason the Triangles AGB , ACH , ADI , AEP , are equi-angular; the Squares of the right Lines AB , AC , AD , AE , will be proportionable to the Squares of the right Lines AG , AH , AI , AP : so that the Deviations BM , CN , DO , EP , will be proportionable to the Squares of the right Lines AG , AH , AI , AP . Let the right Line L be a third Proportional to the right Lines EP , AP ; and it will be [by 17 *EL* 6.] $L \times EP = AP^2$. but $AP^2 : AG^2 :: EP : BM :: L \times EP : L \times BM$; whence since it is $L \times EP = AP^2$. it will be $L \times BM = AG^2$. In like manner, it will be $L \times CN = AH^2$. and $L \times DO = AI^2$. But because it is $BG : AG :: (EP : AP :: \text{by Hypothesis}) AP : L$; it will be $L \times BG = AG \times AP = AG \times AG + AG \times GP = AG^2 + AG \times GP$. But it has been shewn, that

that it is $L \times BM = AG \times q$, wherefore it will be $L \times BG - L \times BM = AG \times GP$, that is, $L \times MG = AG \times GP$. By the same way of Reasoning it will be $L \times NH = AH \times HP$, and $L \times OI = AI \times IP$, as also $L \times VK = AV \times VP$. Wherefore by the preceding Lemma, the Curve $AMNOPK$, wherein the Projectile is moved, will be a Parabola. *Q. E. D.*

Cor. 1. THE right Line L is the *Latus Rectum* or Parameter of the Parabola, that belongs to its Axis.

Cor. 2. LET be $AH = HP$, and it will be $L \times CN = AH \times q = L \times NH$, whence it will be $NH = CN$; and consequently the right Line AF being the Line of Direction of the Projectile, will [by Prop. 33. Lib. 1. of *Apollonius*] touch the Parabola.

Cor. 3. BECAUSE it is $AP = 2AH$, it will be $PE = 2CH = 4CN$ or $4NH$.

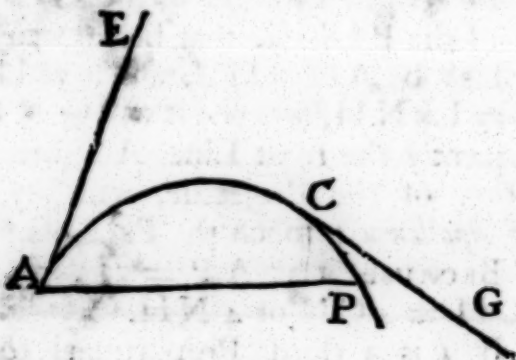
Cor. 4. IF l is a third Proportional to the right Lines PE , AE , l will be the *Latus Rectum*, or Parameter of the Parabola, that belongs to the Diameter AS . For because PE , AE , l , are continual Proportionals; it will be $l \times PE = AE \times q$, that is $AE \times q = AB \times q$ or $QM \times q :: PE : BM$ or $AQ :: l \times PE : l \times AQ$. Wherefore since it is $AE \times q \times PE$, it will be $QM \times q = l \times AQ$: wherefore l will be the Parameter belonging to the Diameter AS .

Cor. 5. BUT it is $l = PE + L = 4NH + L =$ four times the Height of the Parabola $+ L$. For it is $l \times PE = AE \times q = AP \times q + PE \times q = L \times PE + PE \times q = L + PE \times PE$. Wherefore it will be $l = L + PE = L + 4NH$.

Cor. 6. IF the Times AB , BC , CD , are made equal; the Horizontal Spaces AG , GH , HI , will be equal; that is, if a heavy Body, in its Motion, describes a Parabola, in equal Times, it will advance equally according to a Direction that is parallel to the Horizon; and in every Point of the Parabola,

the Horizontal Impetus will remain the same as it was at the beginning of Motion.

Cor. 7. If a Body projected from A in the Direction AE, describes the Parabola ACP; in any Point C it will endeavour, by the first Law of Nature, to run out along the Tangent CG, with all the Velocity that it has in the Point C, and is solely



retained in the Parabolical Curve by the Force of Gravity. But if another heavy Body should be projected from C in the Direction CG, with the Velocity that the Body projected from A, has in the same Point C; that other Body will describe the same Parabola CP. For in the Point C, the Direction, Velocity, and Force of Gravity of both Bodies are the same; wherefore the Way of both will be the same.

Cor. 8. HENCE if a heavy Body is projected downwards, in a Direction oblique to the Horizon; the Path of the Projectile will be a Parabola.

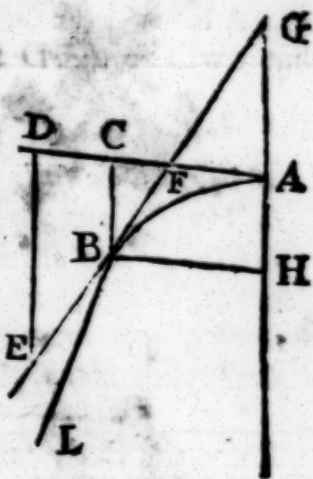
THEOR. XLIX.

THE Impetus of a projected Body in different Points of the Parabola, are as the Portions of the Tangents intercepted betwixt two right Lines parallel to the Axis.

LET a heavy Body describe the Parabola ABL, which let the right Lines AD, BE, touch in the Points

Lect. 16. *Natural Philosophy.* 261

Points A and B. The Impetus of the Body in the Points A and B, will be as CD, EB, the Portions of the Tangents intercepted betwixt two right Lines parallel to the Axis. For if the Body should be deprived of its Gravity in the Point A, it would run out in the Tangent AC, with the same Impetus that it has in the Point A. So likewise the moving Body in B having lost its Gravity, would proceed in the Tangent BE with all the Velocity that it has in the Point B. But in the Points A and B, the Horizontal Impetus remains the same, as appears by Cor. 6. of the preceding Theor. so that the Body in A going out into the Tangent AD, and in B into the Tangent BE, in equal Times it will proceed through equal Spaces according to the Horizontal Direction. Therefore CD in the Tangent AD, and BE in the Tangent BE, will be passed over in equal Times; but the Velocities, or Impetus of the moving Bodies, are as the Spaces passed over in equal Times: wherefore the Impetus of the moving Body in A, is to the Impetus of the same Body in B, as CD to BE. Q. E. D.



Cor. IF A is the Vertex of the Parabola, and the Tangent produced, till it meets the Axis in G; the Impetus in A will be to the Impetus in B, as the Ordinate BH to the Tangent BG: for it is $CD : BE :: CF : BF$ (by reason the Triangles CBF, BHG, are similar): $:: BH : BG$.

Defin. LET ACF be a Parabola, in whose Axis produced beyond the Vertex let be taken $GA = \frac{1}{2}$ the Parameter. *See the Fig. of the following Theorem.*

Draw CB parallel to AD ; and from any Point F of the Parabola, draw FH parallel to AE , and FE to HA : If there was no Gravity, and the Body was projected in the Direction AE with the Velocity that is acquired in falling from G into A , it would be carried in the same Time through the Double of GA ; so that it would describe in that Time $AB = DC = 2GA$. But the Body, on account of its Gravity, beginning to descend in the Point A , will fall in the same Time through the Space $BC = AG$; wherefore in its Motion it will pass through the Point C in the Parabola. Again, let the Body be supposed by its Horizontal Motion (abstracting from that which arises from its Gravity) to arrive in a certain Time at E , beyond or on this side B ; and since the Motion in the Direction parallel to the Horizon remains equable, AB , AE , will be as the Times wherein they are passed over. But the Descents or Deviations of the moving Body from the right Line AE , are as the Squares of the Times wherein they are made: wherefore by reason BC , EF are proportionable to the Squares of the right Lines AB , AE ; since C is the place of the heavy Body at the end of the Time AB , F will be the Place of the same Body at the end of the Time AE ; and so the heavy Body will be always found in the Parabola ACF . *Q. E. D.*

Cor. HENCE the Velocity of a heavy Body in the Vertex of the Parabola which it describes, is that which is acquired by falling from the Sublimity of that Parabola.

LEMMA.

LET BA be a Parabola, whose Axis is AF , Sublimity AG , and Tangent BC , Ordinate BF ; it will be $BFq. : BCq. :: GA : GF$.

FOR it is [by 33 Lib. 1, of *Apollonius's Conicks*], $CF = 2AF$, and from the Nature of the Parabola

$$S 4$$

$$4GA$$



$4 G A \times A F = B F q$. Wherefore it will be $B F q : B C q :: 4 G A \times A F : 4 G A \times A F + C F q :: 4 G A \times A F : 4 G A \times A F + 4 A F q :: G A : G A + A F$ or $G F$. $\mathcal{Q}$. *E. D.*

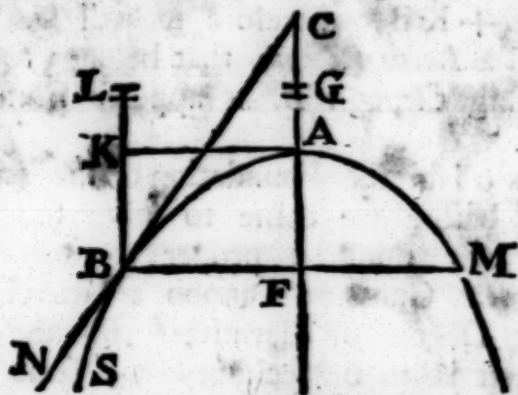
THEOR. LI.

A HEAVY Body being projected directly upwards, with the same Impetus wherewith another heavy Body is projected obliquely, will ascend to a Height equal to the Height and Sublimity, taken together, of the Parabola, which the oblique Projectile will describe in its Motion.

LET from B be projected a heavy Body in the Direction B C, describing in its Motion the Parabola B A M, whose Axis is A F, Vertex A, and Sublimity G A : I say, if the same or another heavy Body be projected from B directly upwards, with the same Impetus, it will ascend to L; so that B L will be equal to F G, the Height and Sublimity of the Parabola taken together. By Cor. of Theor. 49. the Impetus of the heavy Body in B, is to the Impetus of the same Body in A, as B C to B F; but the Impetus acquired by falling from G to F, is to the Impetus acquired by falling from G to A, in a subduplicate Ratio of G F to G A, that is, (by reason it is

B C q.

BCq.:BFq.:GF:GA) as BC to BF. Wherefore the Impetus in B, will be to the Impetus in A, as the Impetus acquired by falling from G to F.



to the Impetus acquired by falling from G to A; but the Impetus of the heavy Body in the Vertex A, is that which is acquired by falling from G to A; wherefore the Impetus or Velocity of the same Body in B, is that which is acquired by falling from G to F, or from L to B, which Height is equal to the Height and Sublimity of the Parabola taken together. But the heavy Body projected directly upwards with the same Impetus, will ascend to L; wherefore if a heavy Body is projected directly upwards with the Impetus that another heavy Body, describing the Parabola B A M, has in the same Point B, it will ascend to a Height equal to the Height and Sublimity of the Parabola taken together.

Q. E. D.

Cor. 1. IF a heavy Body falls from L to B, and the Impetus acquired in the Fall remaining the same, the Direction of Motion should be changed, by Reflection, or the like manner, into the right Line BC, or BN, so that the heavy Body begins to descend again; it will in its Motion describe the Parabola SBAM.

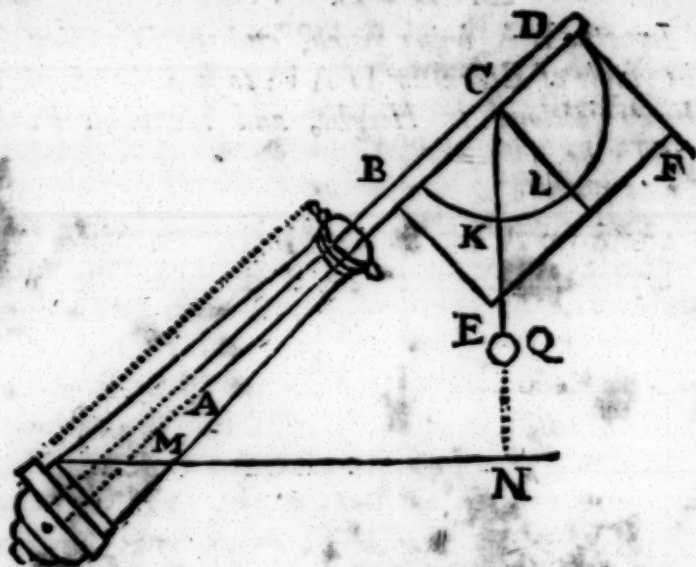
Cor.

Cor. 2. THE Impetus in any Point B of a Parabola, is that which is acquired, by falling through the fourth part of the *Latus Rectum* belonging to the Diameter that passes through that Point. For it is $LB = \frac{1}{4} L + KB$; wherefore it will be $4 LB = L + 4 KB = \text{Latus Rectum}$ that belongs to the Diameter passing through B, as appears from Cor. 5. Theor. 48.

HAVING laid the Foundation of the Doctrine of Projectiles, before we come to solve the following Problems, we think it proper to shew the way, whereby great Guns or Cannon are directed according to any Degree of Elevation. But the Direction of a Cannon is to be reckoned the same with the Direction of its Concavity: for by the firing of the Gun-Powder, the Bullet is emitted according to the Concavity of the Cannon or Mortar; and if it was not for Gravity, it would proceed in that right Line produced: so that that right Line is the Direction of the Ordnance.

WHEREFORE that the Ordnance may be aimed at the Mark, it ought not to be directed according to its outward Metal, since Ordnance are thicker towards their Breech than near their Orifice, because their Resistance ought to be greatest in that part, which suffers most from the Gun-powder: whence, that a Cannon may be the more readily directed, something is added to the Orifice, (which they call *Dispart*) that its Thickness may equal the Thickness of its Breech, and then it is levelled by a Line parallel to the Concavity of the Piece; and after the abovemention'd manner the Ordnance is aimed directly at the Mark. When Walls are to be battered, or any thing to be done, where a great Impetus is required, and the Mark is not above 200 Paces off, and the Engine sufficiently large; in such Shots, besides what has been said, and the Experience

rience of making proper Charges of Powder, and suitable Balls, there is no other Artifice required. But since most commonly the Citadels, or Enemy that is to be fired upon, by reason of the too great distance, cannot be hit by a direct Level; or when the Houses of the City are to be broken down or burnt by the flinging of Bombs; the Engine must be elevated in an Angle inclined to the Horizon: to which end, there is used a Ruler $ABCD$, whereunto is fixed a Parallelogram $BEFD$, on which is inscribed a Semi-circle divided into Degrees, from whose Center hangs a Plummet; but the Extremity of the Ruler A must be inserted into the mouth of the Engine, and be held in a Situation that is paral-



lel to its Axis. And so the Ordnance is to be raised or depressed, till the Perpendicular CQ touches, in the Limb of the Semi-circle, the Point K , namely, the Degree of the Elevation required, reckoning from L towards B ; for it is manifest, that the Angle LCK is equal to the Angle CMN , the Elevation of the Engine, because the Angle $M CN$ is the Com-

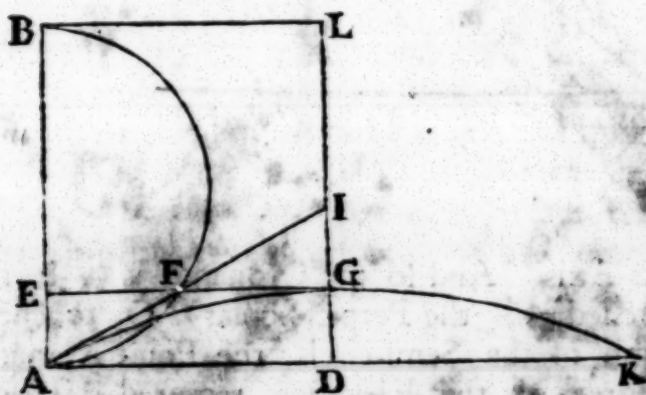
Complement of both to a right Angle. The Parallelogram $BEFD$ is frequently used alone without the Ruler, by applying its Side BE to the Mouth of the Engine, whereby the Perpendicular CQ will shew the Degree of Elevation

Defn. By the Impetus designed by any Perpendicular AB , we understand an Impetus sufficient to hurl the proposed heavy Body from A to the highest Point B of the Perpendicular AB ; or, which is the same thing, the Impetus acquired by falling from B to A ; for by no other way can an Impetus be reduced under any certain and universal Rule, than by determining it after this manner by Spaces.

PROBL. VIII.

THE Impetus BA being given, and the Direction AI , or the Angle of Elevation DAI ; to find the Amplitude of the Projection, its Height, and the whole Path it will describe.

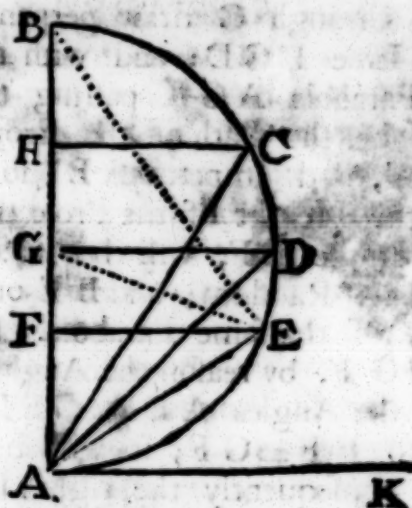
DRAW from A and B the Horizontal Lines AD , BL . Upon the Diameter AB describe the Semi-



circle AFB , which will cut the Line of Direction AI in F : through F draw EF parallel to the Horizon,

zon, and produce it to G, so that it may be $GF = EF$; and also through G draw perpendicular to the Horizon the Line $LG D$; and with the Vertex G describe the Parabola AGK passing through A: I say, this will be the Path of the Projectile, whose Direction is AI , and Impetus AB ; so that DG or AE will be the Height of the Projection. Double AD , or quadruple EF , will be its Amplitude or whole Horizontal Random; and BE or LG will be the Sublimity of the same Parabola. In the Triangles AEF , IGF , by reason the Angles at E and G are right, and the Angles AFE , GFI , at the Vertex equal, also $EF = GF$; it will be $IG = AE = DG$, and consequently the right Line AI will touch the Parabola. And because it is $AD = EG = 2 EF$; it will be $ADq. = 4 EFq. = 4 BE \times EA = 4 LG \times GD =$ Rectangle under the *Latus Rectum* and GD : wherefore it will be $4 LG =$ the *Latus Rectum* of the Parabola, whence LG will be the Sublimity of the same Parabola. Wherefore [by Cor. Theor. 51.] if the heavy Body falls from B to A, and is projected in the Direction AI with the Impetus acquired in falling, it will describe the Parabola AGK .

Cor. 1. HENCE it is manifest, from the Impetus AB of any Engine being given, about which is described the Semi-circle ADB , that the Heights and Amplitudes of all Projectiles, which can be made by the same Engine, are given. For example, the Impetus AB remaining always the same, the Projection made in the Direction AE , has its Height AF , and its Amplitude the Quadruple of EF ; in like manner, it being made in the Direction AD , its Height will be AG , and its Amplitude the Quadruple of GD ; and so of the rest. Whence if the Angle of Elevation DAK is half a right one, the Quadruple of GD will be the greatest of all the Amplitudes, which
can



can be made with the same Impetus; and the Amplitudes of Projections, which are equally distant, from the Projection whose Elevation is half a right Angle, are equal: as, for example, the Projection in the right Lines A E, A C, (the Angles D A E, D A C, being made equal) have their Amplitudes quadruple of E F, and quadruple of H C, which are equal. Besides, the Amplitude of the Projection whose Elevation is half a right Angle, is $= 4 G D = 4 G B =$ *Latus Rectum* of the Parabola. But the perpendicular Projection upwards, that is, the Impetus of the Projection, will be equal to half the Amplitude of the Protection, elevated to half a right Angle, and made with the same Impetus. Lastly, to make equal Randoms in a Horizontal Plane, there is required a less Impetus in the Projection whose Elevation is half a right Angle: for if it is not less than the Impetus of another Projection, made in another Direction; the Amplitude of the Projection whose Elevation is half a right Angle, will be greater than the Amplitude of that other Projection.

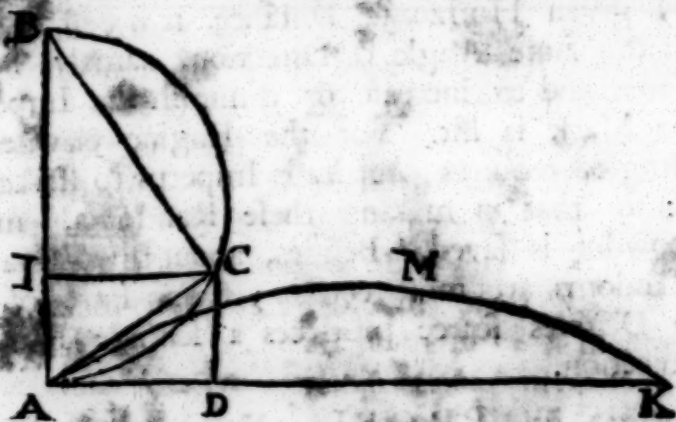
Lect. 16. *Natural Philosophy.* 271

Cor. 2. BECAUSE AK touches the Circle, it will be [by 32 El. 3.] the Angle $ABE = EAK$ the Angle of Elevation; and consequently, the Angle AGE is double EAK. Wherefore GA, half the Impetus, being made Radius, EF the fourth part of the Amplitude, will be the Sine of double the Angle of Elevation; and AF, the Height of the Projection, will be the versed Sine of the Arch AE, or of double the Angle of Elevation; and FB, the Sublimity of the Parabola, will be the versed Sine of the Arch BE, or of the Complement of double the Angle of Elevation to two right Angles.

PROBL. IX.

THE Amplitude AK, and the Angle of Direction CAK, being given; to find the Impetus of the Projection, and the Height AI.

TAKE AD the fourth part of the Amplitude; and erect the Perpendiculars DC, AB; and make the Angle ACB a right one: I say, AB will be the



Impetus of the Projection, and DC will be its Height. For because the Angle ACB is a right one, the Semi-Circle described on the Diameter AB, will pass through C; whence [by Corol. 1. of the preceding

ceding Problem] the Projection whose Direction is AC , Impetus AB , will describe in its Motion the Parabola AMK , whose Height is DC or AI , and the fourth part of its Amplitude is AD ; wherefore conversly the Projectile whose Direction is AC , fourth Part of its Amplitude AD , will have the Impetus AB , and Height DC . *Q. E. D.*

Cor. 1. HENCE from any Horizontal Random of any Engine being given, and made at a given Elevation; may be found the Height to which a Ball will rise, being shot directly upwards: to wit, the Impetus of the Engine, which in great Ordnance exceeds any perpendicular Height to which any Man can come at. But the Impetus being given, the Amplitude and the Height of a Random made from any Elevation, will be given: whence may be discovered, whether a Mark, whose Distance is known, can be hit by a given Engine.

Cor. 2. IF AD , the fourth part of the Amplitude, is made Radius, the Height DC will be the Tangent of the Angle of Elevation. To strike a Mark at any given Horizontal Distance, it is best to keep always the same Angle of Direction, namely, half a right one, and to increase or diminish the Impetus, till the Mark is hit. For the Engine elevated to this Angle, requires the least Impetus to strike the Mark: so that in making these Randoms, much Gun-powder is saved. Besides, about this Elevation the Randoms are most certain; for an Error of one or two Degrees scarce produces a sensible Error in the Projection.

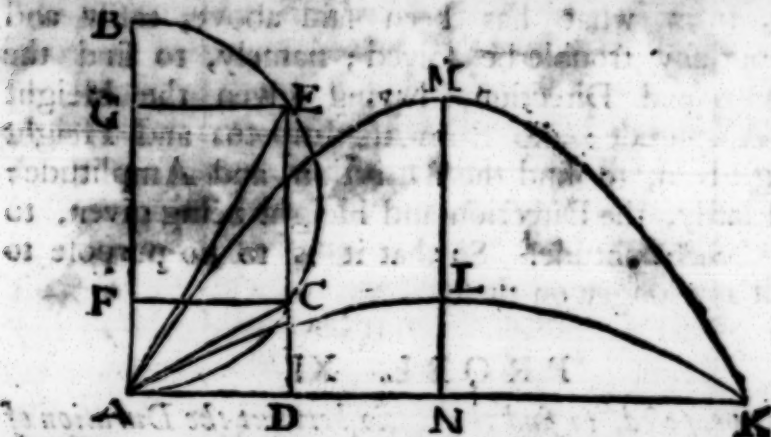
PROBL. X.

THE Impetus and Amplitude being given, to find the Direction and Height of the Random.

LET AB be the Impetus, and AD the fourth part of the given Amplitude; upon the Diameter AB

Lect. 16. *Natural Philosophy.* 273

AB describe a Semi-circle ACEB, and erect the Normal DCE, cutting the Semi-circle in the Points C and E: I say, both the Directions, whether AC



or AE, will describe a Parabola, whose Amplitude will be AK quadruple of AD. For the Projections made with the Impetus AB, in the Directions AC or AE, have the Amplitude AK quadruple of FC or GE, [by Probl. 8.] but the Height may be either AF or AG, as is manifest. But if the Normal DC meets the Circle in one Point only, that is, touches it; there will be but one Parabola described, and that by a Projection whose Elevation is half a right Angle; and the proposed Amplitude will be the greatest possible with that Impetus. If the Perpendicular DC does not meet the Semi-circle, the Problem will be impossible.

Cor. I. If the Impetus of any Engine is known, (as it may be found by Corol. 1. of the preceding Probl. from any Horizontal Random) by the help of this Problem; such an Elevation may be given to the Engine, that the Mark placed at any given Horizontal Distance may be struck; and of two Directions fit for the purpose, equally remote from the Direction whose Elevation is half a right Angle; the more proper may be made choice of.

T

SGHQ

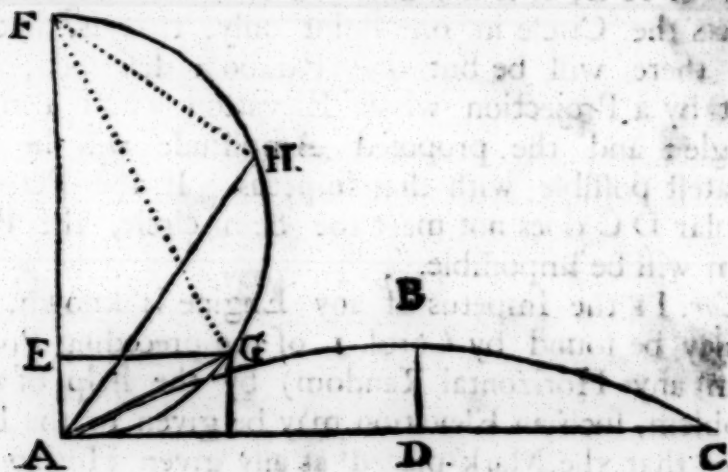
S C H O L I U M.

THE Converse of the three preceding Problems, may, from what has been said above, easily and without any trouble be solved; namely, to find the Impetus and Direction, having given the Height and Amplitude: also from the Impetus and Height being given, to find the Direction and Amplitude: And lastly, the Direction and Height being given, to find the Amplitude. So that it is to no purpose to dwell any longer on these.

P R O B L. XI.

IT is proposed, to find the Ratio betwixt the Duration of a Projection made perpendicularly upwards, and of any other, whose Impetus is the same.

LET AF be the Impetus of the Projectile, or the Projection made upwards, and ABC the Projection from any other Elevation AG; about the Diameter



AF describe a Semi-circle, cutting the Direction AG in G: I say, the Duration of the Projection directly upwards, or the Time of Ascent through AF,

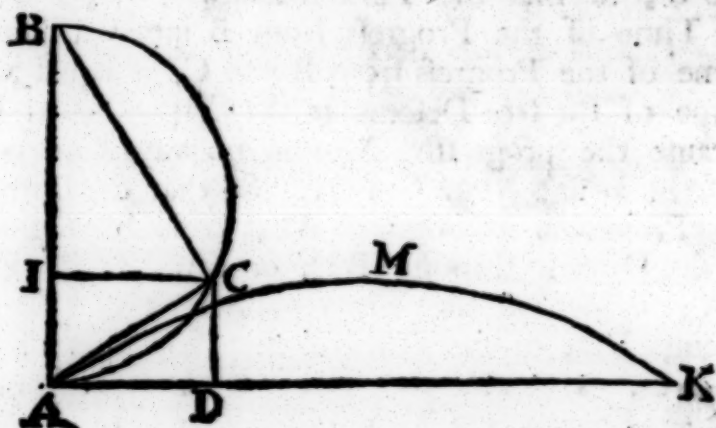
AF, and of Descent through the same, is to the Duration of the Projection in the Parabola ABC, as AF to AG. The Time of the Progress from A into B, is equal to the Time of the Progress from B into C; so that the Time through ABC, is double the Time of the Progress from B into C; but the Time of the Progress from B into C, is equal to the Time of the free Descent in the Perpendicular BD, because the progressive Motion no ways hinders the Descent arising from Gravity. So that the Time of the Projection through ABC, is double the Time of the Descent through BD, or through its Equal EA; so likewise the Time of Ascent and Descent through FA, or the Time of the Projection directly upwards, is double the Time of Descent through FA; wherefore the Time of the Projection upwards, will be to the Time of the Projection in the Parabola ABC, as the Time of Descent through FA, to the Time of Descent through EA; that is, in a subduplicate Ratio of FA to EA; or, by reason FA, AG, EA, are continual Proportionals, as FA to AG. Q. E. D.

Cor. THE Durations of Projections made with an equal Impetus in different Directions AG, AH, are in the Ratio of the Chords AG, AH. And if AF is made Radius, AG will be the Sine of the Angle AFG, which is equal to the Angle of Elevation of the Engine: so that the Time of the Projection directly upwards, is to the Time of Projection in a Parabola, as Radius to the Sine of the Angle of Direction.

S C H O L I U M.

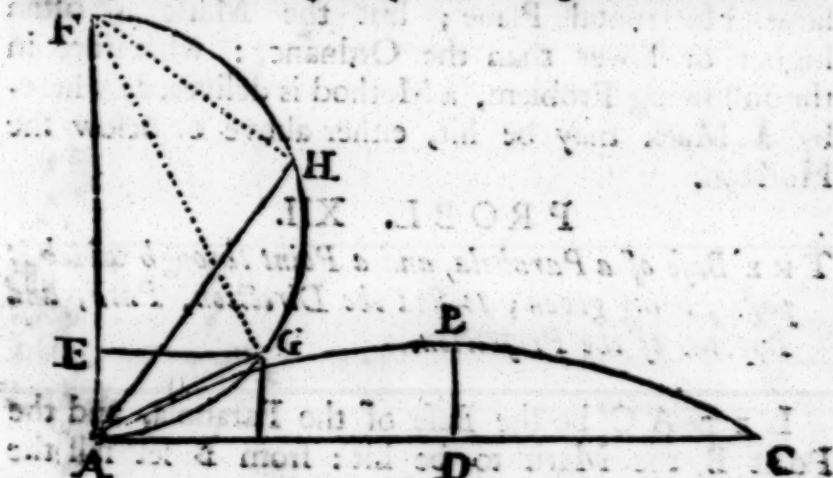
ALL the Problems about the Projections of heavy Bodies made in a Horizontal Plane, are easily resolved by the means of a Table of Sines and Tangents.

LET AK be the Horizontal Amplitude of any large Piece of Ordnance, elevated to the given Angle CAK , there is required the Height of the Projection, and the Impetus of the Engine. In the



Triangle ADC let it be as the Radius to the Tangent of the Angle of Elevation; so is AD , the fourth part of the given Amplitude, to the Height DC . Also make as the Sine of the Angle of Elevation to Radius, so the Height found DC to AC , which therefore will be given; and in the rectangle Triangle BCA , make as the Sine of the Angle ABC (which is equal to the Angle of Elevation) to Radius, so AC to AB the Impetus, which consequently is known. But the Impetus being given, the Time of the perpendicular Projection will be likewise given. But the Time of the perpendicular Projection is to the Time of the Projection in AC , as AB to AC , or as Radius to the Sine of the Angle of Elevation; and consequently the Time of Projection in AC may be known from a Table of Sines. Hence also from the Time of any Projection being given, made according to a given Elevation, will be given the Time of any other Projection, made with the same Impetus. For as the Sine of Elevation of a Projection,

jection, whose Time is known, is to the Sine of another Elevation; so is the Time known of one Projection, to the Time of the other Projection, which will therefore be also known. But from the Amplitude of one Projection being given, made in a given Direction, will be given the Amplitude of a Projection made in any other Direction. For making half the Impetus Radius, the fourth part of the Amplitude will be the Sine of double the Angle of Elevation; and consequently the Amplitudes are as the



Sines of these Angles. Wherefore if the Amplitude in the Direction AG is known, the Amplitude in the Direction AH will be given: for make as the Sine of double the Angle CAG , to the Sine of double the Angle HAC , so the Amplitude of the Projection according to AG , to the Amplitude of the Projection according to the Direction AH ; and if from the Impetus and Horizontal Amplitude being given, there is sought the corresponding Elevation, that may be easily discovered from the same Principle. For it is manifest from Cor. 2. Probl. 8. that double the Impetus is the Amplitude of the Projection whose Elevation is half a right Angle. But the Sines of the Elevations doubled are as the Am-

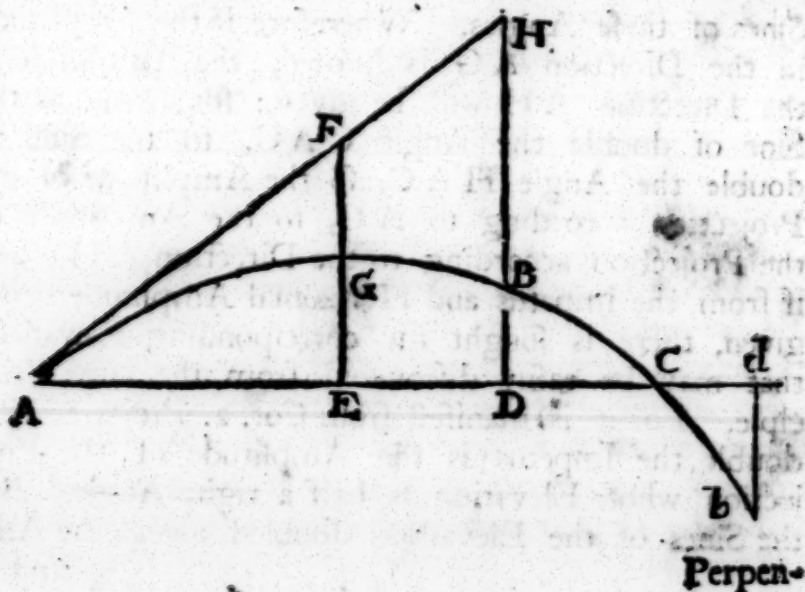
plitudes; wherefore make as double the Impetus to the given Amplitude, so the Sine of double half a right Angle, that is, the Sine of ninety Degrees, or Radius to another; which will be the Sine of two Arches, whereof one is the Complement of the other to a Semicircle: and these two Arches halved, will give two Elevations, wherewith the given Amplitude may be reached.

THE warlike Engines are not always to be so exploded, as the Bullet shall fall precisely in the same Horizontal Plane; but the Mark is often higher or lower than the Ordnance: wherefore in the following Problem, a Method is delivered, whereby a Mark may be hit, either above or below the Horizon.

PROBL. XII.

THE Base of a Parabola, and a Point through which it passes, being given; to find the Direction, Path, and Impetus of the Projection.

LET AC be the Base of the Parabola, and the Point B the Mark to be hit: from B let fall the



Perpendicular BD on AC ; take L a fourth Proportional to the right Lines BD , AD , DC ; L will be the *Latus Rectum* of the Parabola. Bisect AC in E , and from E erect the Perpendicular EF ; let EG be a third proportional to the right Lines L and AE ; G will be the Vertex of the Parabola: and if EG is produced, so that it is $GF = GE$, and AE be drawn, FAE will be the Angle of Direction of the Engine. And the Impetus wherewith the heavy Body is to be projected, will be equal to $EG + \frac{1}{2}L$. Because BD is to AD as DC to L ; it will be $L \times BD = AD \times DC$; so that [by Cor. Theor. 48.] L is the *Latus Rectum* of the Parabola, passing through B , whose Base is AC . And because L , AE , EG , are Proportionals, it will be $L \times EG = AE^2$. so that G will be the Vertex of the Parabola. Therefore with the Vertex G , and *Latus Rectum* L , a Parabola being described, it will be the Path of the Projection of the heavy Body, that will strike the Point B . And the Impetus of the Projection is equal to $EG + \frac{1}{2}L$: but the Angle of Elevation is FAE .
Q. E. I.

You must proceed after the same manner, if the Point b is below the Horizon; for if from b be let fall the Perpendicular bd on AC produced, and L is taken a fourth Proportional to bd , Ad , dC , L will be the *Latus Rectum* of the Parabola passing through b .

Cor. AE being made Radius, EF or the Double of EG will be the Tangent of the Angle of Elevation; so that if it be made as AE , which is given, to EF also given, so Radius to the Tangent of the Angle FAE , the Angle of Elevation will be given.

PROBL. XIII

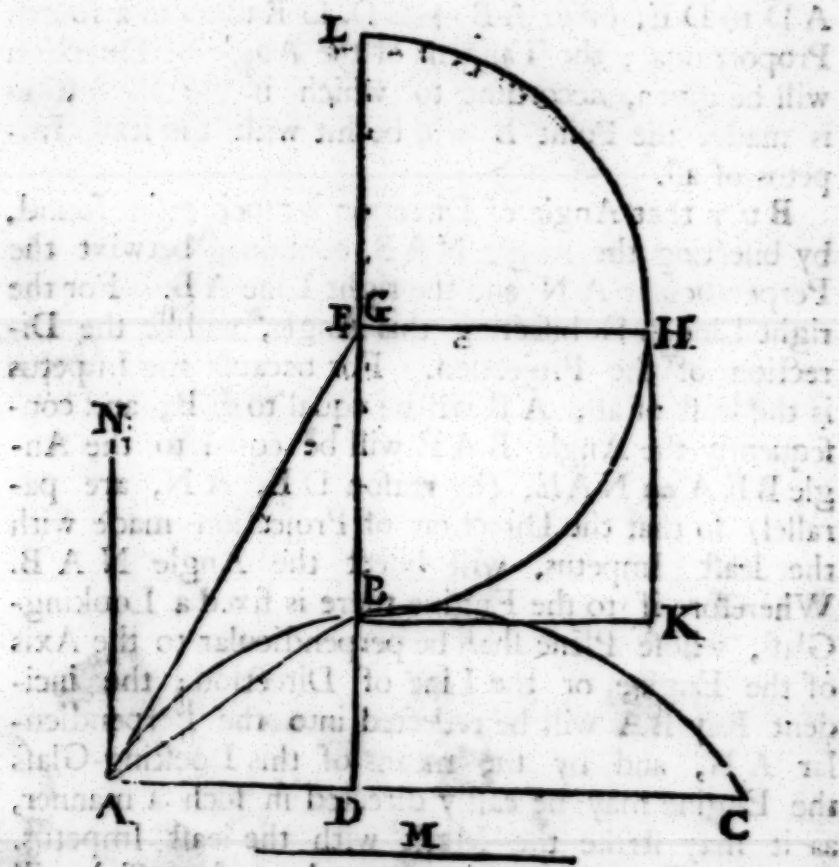
THE Impetus being given, to find the Direction according to which a heavy Body being projected, that Body may hit a given Point.

Lect. 16. Natural Philosophy.

287

It will be $4 M \times EB = (ADq. + BDq. + 2 BD \times EB + EBq. = ADq. + DEq. =) AEq.$ But the Parabola described by a heavy Body, projected in the Direction AE , with the Impetus M , will so cut the right Line DE , that it is $4 M \times EB = AEq.$ (as is manifest from Cor. 2. Theor. 51.) Wherefore the Point B is in the same Parabola; and the heavy Body projected with the Impetus M , in the Direction AE will pass through B . Q. E. D.

Cor. If HK meets the Circle in one Point only, that is, if it touches the Circle, there will be only



one Direction that will satisfy the Condition. But if it does not meet the Circle at all, the Problem will

will be impossible; that is, the Point B cannot be hit with the given Impetus. And if KH touches the Circle, that Impetus will be the least of all, wherewith the given Point can be hit. And in that case it will be BK or $AB = BE$ or $BG = 2M - DB$; so that it will be $BE + BD$ or $DE = 2M$. The least Impetus therefore, wherewith a given Point may be hit, will be equal to half $DE = \frac{AB + BD}{2}$; and DA being made Radius, DE will

be the Tangent of the Angle EAD, that is, of the Angle of Elevation. Wherefore if it be made as AD to DE, or to $AB + BD$, so Radius to a fourth Proportional; the Tangent of the Angle of Direction will be given, according to which if the Projection is made, the Point B will be hit with the least Impetus of all.

But that Angle of Direction is much easier found, by bisecting the Angle NAB, contained betwixt the Perpendicular AN and the right Line AB. For the right Line AE bisecting this Angle, will be the Direction of the Projection. For because the Impetus is the least of all, AB will be equal to EB; and consequently the Angle BAE will be equal to the Angle BEA = NAE, (by reason DE, AN, are parallel) so that the Direction of Projection made with the least Impetus, will bisect the Angle NAB. Wherefore if to the Engine there is fixed a Looking-Glass, whose Plane shall be perpendicular to the Axis of the Engine or the Line of Direction; the incident Ray BA will be reflected into the Perpendicular AN, and by the means of this Looking-Glass the Engine may be easily directed in such a manner, as it may strike the Mark with the least Impetus. For the Engine is to be elevated or depressed, till the Image of the Point B, made on the Plane of the Looking-Glass, may be seen in the Perpendicular NA;

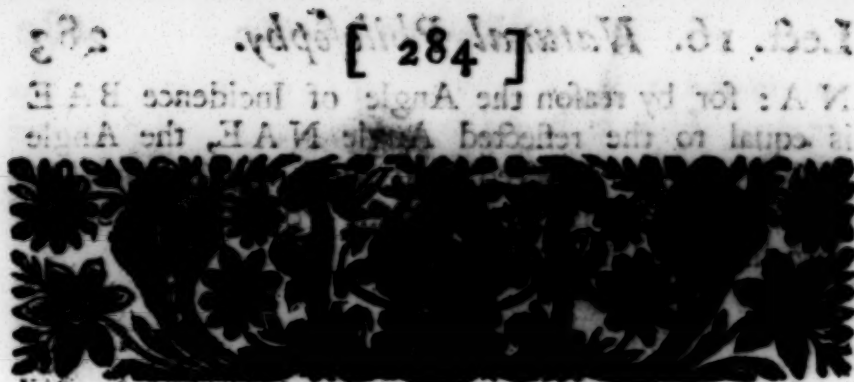
Lect. 16. *Natural Philosophy.*

283

NA: for by reason the Angle of Incidence BAE is equal to the reflected Angle NAE, the Angle NAE will be bisected, and AE will be the Diameter of the Engine, when the Point E may be hit with the last Lenses.



THE



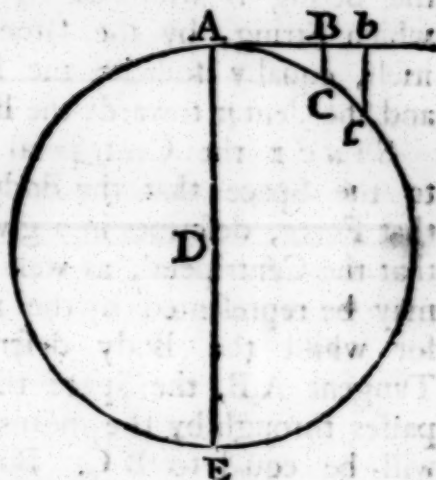
THE
THEOREMS
Of the Famous
Monf. *HUGENS*,
CONCERNING
The CENTRIFUGAL FORCE, and
CIRCULAR MOTION,
DEMONSTRATED.

 *FIRST* communicated the Demonstrations of the following Theorems to the Learned World; for their Author published them without any Demonstration. But afterwards the same Theorems were demonstrated by some French-men, but not in the same Order; and now at length the Author's own Demonstrations are published in his Posthumous Pieces, which though they are indeed very short, yet they are more prolix than ours. But since these Theorems make no mean Part of the Science of Motion,

we

we have thought fit to join their Demonstrations to this Work; that the Commonwealth of Learning may see how much the Mechanical Philosophy is capable of being improved by Geometry.

Defn. 1.  **CENTRIPETAL** Force is that Force, whereby any moving Body is continually drawn from its rectilinear Motion, and is perpetually sollicitated towards some Center. For since, according to a well-known Law of Nature, every Body once in Motion endeavours to proceed always uniformly in the same right Line, it is manifest no moving Body can describe an Orbit in its Motion, unless it is detained in that Orbit by some Force. For example, let a moving Body revolve with a uniform Motion in the Circumference of the Circle *A C E*, which when it comes to *A*, if the Force that kept it in the Orbit is taken away, it will proceed along the Tangent *A B*, and run out *in infinitum*: that it



therefore may be detained in the Circumference, it is necessary that some Force continually acts, and which is equivalent to the Force in *A* solliciting the Body towards *D* through a Space equal to *B C*, in the mean time whilst the moving Body by its innate Force proceeds through the infinitely small Space *A B*: For after this manner, by these Forces joined together, the moving Body will describe the Line *A C* [by Theor. 30.] This Force, whether it is the Action of a

String

String holding the Body, or its Coherence with another revolving Body, or whether it arises from Gravity or any Attraction whatsoever, it may be called a Centripetal Force.

2. A CENTRIFUGAL Force is the Re-action or Resistance which a moving Body exerts to prevent its being turned out of its way, and whereby it endeavours to continue its Motion in the same Direction: and as Re-action is always equal, and contrary to Action, so in like manner is the Centrifugal to the Centripetal Force. This Centrifugal Force arises from the *Vis Inertia* of Matter, and when a Body revolving in the Circumference of a Circle, is detained by the means of a String from running out; the String is stretched by this Centrifugal Force: which String, by the same Endeavour of relaxing itself, equally sollicit the Body towards the Center, and the Center towards the Body.

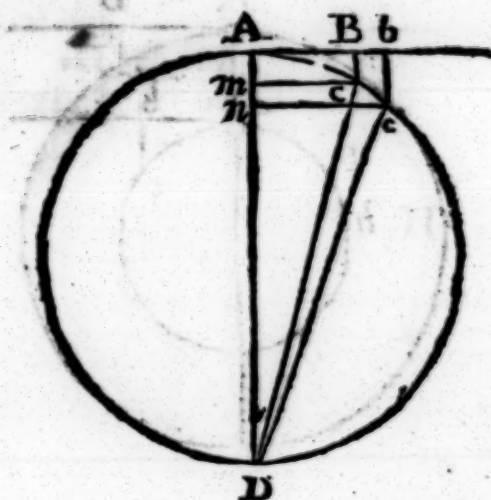
SINCE the Centripetal Force is proportionable to the Space that the Body, by the Sollicitation of that Force, describes in a given Time; it is manifest, that the Centripetal, as well as the Centrifugal Force, may be represented by the nascent Lines BC or bc : for whilst the Body describes the infinitely small Tangent AB , the Space that in the mean time it passes through by the means of the centripetal Force, will be equal to BC . But we have demonstrated (Lect. 4.) in nascent or infinite small Lines AB , AC , that BC is infinitely less than AB or AC . Whence the Centripetal or Centrifugal Force will be infinitely less than the communicated Force AB .

LEMMA I.

In a Circle the evanescent or infinitely small Subtenses of the Angle of Contact, are in a duplicate Ratio of the conterminal Arches.

LET

LET the Arches be AC , Ae , the Subtense perpendicular to the Tangent BC , Bc ; draw the Diameter AD , and Om , en , Perpendiculars to the Diameter; and it will be $BC:bc::Am:An::Am \times$



$AD:Am \times AD$. But it is [by 8 *El.* 6.] $AD:AC::AC:Am$, and $AD:Ae::Ae:An$; wherefore it will be $AD \times Am = ACq$, and $AD \times An = Ae^2$; wherefore it is also $BC:bc::ACq:ACq$. *Q. E. D.*

Cor. HENCE it is $BC = \frac{ACq}{DA}$.

THIS Lemma the Great Sir Isaac Newton has universally demonstrated in all Curves of the first Order.

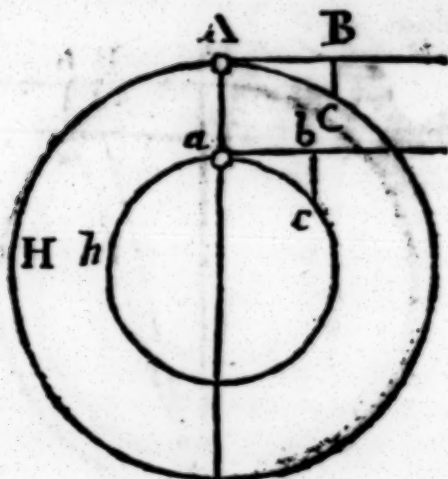
THEOR. I.

IF two equal Bodies in Motion run through unequal Circumference in equal Times; the Centrifugal Force in the greater Circumference, will be to that in the less, as the Circumferences amongst themselves, or their Diameters.

LET the Body in motion A run through the Circumference ACH , and in the same Time let the Body

288 *M. Huygens's Theorems, &c.*

Body a run through the Circumference acb ; and let AC, ac be the infinitely small Arches described in the same Time. Because both Circumferences are



run through in an equal Time, these Arches will be similar, and consequently the Figure ABC will be similar to the Figure abc ; wherefore $BC:bc::AC:ac::$ Circumference ACH : to Circumference acb . But it is manifest from the foregoing Definition, that the Centrifugal Force of the Body A is to the Centrifugal Force of the Body a , as BC to bc . Wherefore the Centrifugal Force of the Body A will be to the Centrifugal Force of the Body a , as the Circumference ACH to the Circumference acb , or as the Diameter of that to the Diameter of this. *Q. E. D.*

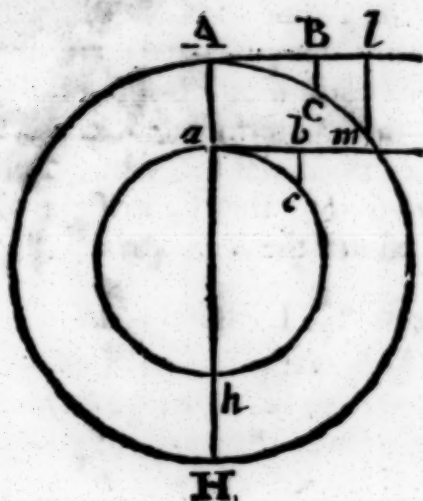
Cor. HENCE conversly, if the Centrifugal Forces are as their Diameters, the Periodical Times will be equal.

THEOR. II.

If two equal Bodies in Motion are carried in unequal Circumferences with an equal Celerity, their Centrifugal Forces will be in a reciprocal Ratio of the Diameters.

LET

LET AC , ac , be the least Arches described in the same Time, which, by reason the Velocity in both the moving Bodies is equal, will be equal. Make



the Arch Am similar to the Arch ac , and draw lm parallel to BC ; and the Centrifugal Force in the greater Circumference will be to that which is in the less, as the nascent Line BC to the nascent Line bc : but BC is to bc in a Ratio compounded of BC to lm and lm to bc ; and from the preceding Lemma, BC is to lm , as ACq to Amq , and lm is to bc as Am to ac or AC . Wherefore it will be $BC : bc :: ACq : Amq + Am : ac :: ACq : Amq + Amq : Am \times ac :: ACq$ or $acq : Am \times ac :: ac : Am$; that is, as the whole Circumference acb to the whole Circumference ACH , or as the Diameter ab to the Diameter AH . Q. E. D.

THEOR. III.

IF two equal moving Bodies are carried in equal Circumferences, but both with an equable Motion, (as we would be understood to mean in all) the Centrifugal Force of
U
the

290 *M. Huygens's Theorems, &c.*

the swifter will be to the Force of the slower in a duplicate Ratio of the Celerities.

FOR the Centrifugal Forces are as the evanescent Subtenses of the Angle of Contact, which (by what we have already demonstrated) are in the same or equal Circles in a duplicate Ratio of the conterminal Arches; but the conterminal Arches, since they are the Spaces described in the same Time, are as the Velocities: wherefore the Centrifugal Forces are in a duplicate Ratio of the Velocities. *Q. E. D.*

THEOR. IV.

IF two equal Bodies in motion, carried in unequal Circumferences, have their Centrifugal Forces equal; the Time of the Revolution in the greater Circumference, will be to the Time of the Revolution in the less, in a subduplicate Ratio of the Diameters.

LET AC , ac , be the least Arches described in the same Time; (*see Fig. Theor. 2.*) because the Centrifugal Forces are equal, it will be $BC = bc$. Let the Time wherein the Circumference ACH is described, be called T , and the Time wherein the Circumference $acba$ is described, be t : make the Arch Am similar to the Arch ac , and let us suppose any moving Body to run through the Circumference $ACHA$ in the same Time, as it would run through the Circumference $acba$; and in that Case the Arches described in the same Time in both Circumferences will be Am , ac . But the Velocity of a moving Body passing through the Arch Am in any given Time, is to the Velocity of a moving Body passing through the Arch AC in the same Time, as the Arch Am to the Arch AC : so that since the Time wherein the same Circumference is run over, is always reciprocally as the Velocity, it will be $T : t :: Am : AC$,
and

and $T^2 : t^2 :: Amq : ACq :: ml : BC :: ml : bc$; that is, by reason the Arch AM is similar to the Arch ac , as the Diameter AH to the Diameter ab : whence it is manifest, that $T : t :: \sqrt{AH} : \sqrt{ab}$. Q. E. D.

Schol. SINCE in every Case the Centrifugal Force is to the Centrifugal Force as BC to bc , but it is $BC = \frac{ACq}{AH}$ and $bc = \frac{acq}{ab}$; the Centrifugal Force

will be to the other Centrifugal Force as $\frac{ACq}{AH}$ to

$\frac{acq}{ab}$: that is, as the Squares of the Arches described in the same Time applied to the Diameters of the Circles. And since those Arches are as the Velocities, the Centrifugal Forces will be as the Squares of the Velocities applied to the Diameters of the Circles.

LEMMA II.

If a moving Body revolves in the Circumference of a Circle, the Space, which the moving Body proceeding forwards, and only the Centrifugal Force arising from that circular Motion taking place, would run over in a given Time, will be a third Proportional to the Diameter of the Circle and that Arch, which if it was carried in the Circumference of the Circle it would describe in the same Time.

LET AC be any Arch described in any the least Particle of Time, and let n denote any Time or any Number of those Particles, $n \times AC$ will be the Arch which the moving Body carried in the Circumference will describe in the given Time n , and BC the Space which it would run over in the first Particle of that Time, the Centrifugal Force taking place. But since every Body in motion, by the same Force

292 *M. Huygens's Theorems, &c.*

continued always in the same Direction, describes Spaces in a duplicate Ratio of the Times, [by Cor. 3. Theor. 12. Lect. 11. for whatever is demonstrated of Gravity, may be applied to any other Force acting uniformly] the Space described, the Centrifugal Force acting, in the Time n , will be $= n^2 \times BC$. But (as is manifest from the first Lemma) it is $AH : AC :: AC : BC$, and as AC to BC , so $n \times AC$ to $n \times BC$. Wherefore AH is to AE as $n \times AC$ to $n \times BC$; and by multiplying the Consequents by n , AH will be to $n \times AC$ as $n \times AC$ to $n^2 \times BC$: that is, the Diameter of the Circle, the Arch described in a given Time, and the Space which, the Centrifugal Force acting, is run over in the same Time, are continual Proportionals. *Q. E. D.*

Cor. If the Diameter of the Circle is called D , and the Arch described by the moving Body, in any Time A , the Space which the moving Body, the Centrifugal Force acting, proceeding forwards would describe in the same Time, will be $\frac{A^2}{D}$; for D , A ,

$\frac{A^2}{D}$, are continual Proportionals.

THEOR. V.

IF a moving Body is carried in the Circumference of a Circle, with the Celerity which it has acquired by falling from a Height that is equal to a fourth Part of the Diameter, it will have a Centrifugal Force equal to its Gravity; that is, it will stretch the String, whereby it is kept to the Center, just as much as if it was suspended by it.

LET the Diameter of the Circle be called D , and the Circumference P : and since, by Hypothesis, the Velocity of the moving Body carried in the Circumference is uniform, and equal to that which it acquires

acquires by falling through $\frac{1}{2} D$, it is certain that the moving Body carried in the Circumference will describe in an equal Time an Arch equal to twice that [by Theor. 12. Lect. 11.] that is, $= \frac{1}{2} D$; whence [by Lemm. 2.] the Space passed over in the mean while, by the acting of the Centrifugal Force, will be $= \frac{1}{2} D$. For D is to $\frac{1}{2} D$, as $\frac{1}{2} D$ to $\frac{1}{2} D$: but by Hypothesis the Space that the moving Body by the Force of Gravity describes in the same Time, is also $\frac{1}{2} D$. Wherefore since the Spaces run over by these two Forces in the same Time, are equal, the Forces themselves will be likewise equal.

Cor. 1. HENCE conversly, if a moving Body carried in the Circumference of a Circle has a Centrifugal Force equal to its Gravity, its Velocity is that which is acquired by falling through $\frac{1}{2} D$.

Cor. 2. HENCE the Time of a Revolution is to the Time of Descent through $\frac{1}{2} D$, as P to $\frac{1}{2} D$, or as $2 P$ to D . For in the Time a moving Body, with an accelerated Velocity, runs over $\frac{1}{2} D$, it will, moved uniformly with the Velocity acquired at the last, run through $\frac{1}{2} D$: and consequently since the Velocities are equal, the Times will be as the Spaces run over; that is, the Time wherein the moving Body runs through the Circumference, is to the Time wherein it describes $\frac{1}{2} D$, as P to $\frac{1}{2} D$, or as $2 P$ to D ; but the Time wherein $\frac{1}{2} D$ is described, is $=$ Time of the Fall through $\frac{1}{2} D$: whence the Time of the Revolution will be to the Time of the perpendicular Fall through $\frac{1}{2} D$, as $2 P$ to D .

THEOR. VI.

IN the concave Superficies of a Parabolick Conoid, which has its Axis erected perpendicularly, all the Revolutions of a moving Body running through the Circumferences parallel to the Horizon, whether they are small or great,

294 *M. Huygens's Theorems, &c.*

are performed in equal Times; each of which Times are equal to two Oscillations of a Pendulum, whose Length is half the Latus Rectum of the generating Parabola.

LET HGADE be a Parabolick Conoid, whose Axis erected perpendicularly is AP; GD, HE, the Diameters of Circles, whose Circumferences parallel to the Horizon the moving Body runs through; which therefore will be sollicitated by three Powers mutually equivalent to each other, acting in three different Directions: whereof the first is the Force of Gravity impelling the moving Body in the right Line HN perpendicular to the Plane of the Horizon; the second is the Centrifugal Force arising from the circular Motion, solliciting the moving Body from H towards K; but the Place of the third Power is supplied by the Resistance or contrary Endeavour of the Parabolick Superficies, according to the Line HP acting perpendicularly to it; for Reaction is always equal to Action, and towards the contrary Part. Whence since the Superficies is pressed perpendicularly by the moving Body, it re-acts on the Body according to the Direction HP, and that contrary Endeavour is equivalent to the Power solliciting the moving Body according to the Direction HP. Wherefore since the moving Body is sustained by these three Powers, they must be necessarily in *equilibrio*, that is, any two destroy the Effect of the other. Whence ON being drawn parallel to HK meeting HN in N, if OH represents the Re-action of the Parabolick Superficies, the right Line ON will expound the Centrifugal Force, and HN the Force of Gravity of the moving Body: But by reason the Triangles HON, HMP, are equi-angular, ON is to HN as HM to MP; that is, the Centrifugal

296 *M. Huygens's Theorems, &c.*

LET the Circle GLD be so taken, that its Diameter GD be equal to the *Latus Rectum* of the Parabola HAE; whence from the Nature of the Parabola, it will be $GB = BQ$: So that the Centrifugal Force of the moving Body in the Circumference GLD will be equal to the Force of Gravity, Therefore [by the preceding Corol.] the Velocity of the moving Body in the Circumference GLD, is that which is acquired by falling through a Space equal to $\frac{1}{2}$ GD, or (from the Nature of a Parabola) through BA. Now let OST be a Cycloid, whose Axis or the Diameter of the generating Circle SR is equal to AB, and the Time of Descent through the Cycloid OS will be to the Time of the perpendicular Fall through the Axis RS, or through BA, as $\frac{1}{2}$ P to D, by Theor. 46. Lect. 15. But [by the preceding Corol.] the Time of Descent through AB is to the Time of the Revolution in the Circumference GLD as D to 2 P; wherefore, by Equality, the Time of Descent through the Cycloid OS, is to the Time of a Revolution in the Circumference GLD as $\frac{1}{2}$ P to 2 P, or as 1 to 4: whence the Time of four Descents in a Cycloid, or the Time of two Oscillations in a Cycloid, is equal to the Time of a Revolution in the Circumference GLD. But the Time of two Oscillations in a Cycloid, is equal to the Time of two the least Oscillations in a Circle, which is of the same curvity with the Cycloid at the Vertex S; by reason a Portion of such a Circle and a Portion of the Cycloid nearly coincide at the Vertex S; and consequently in Physical Matters perform the same Effect, as is now well known. But the Radius of a Circle of the same degree of curvity with the Cycloid at the Vertex S, is equal to twice RS, or twice AB, (as easily follows from Corol. Theor. 46. Lect. 15.) So that the Length of a Pendulum

dulum oscillating in that Circle, is equal to twice AB , or to half the *Latus Rectum* of the generating Parabola. Whence the Time of two the least Oscillations of a Pendulum, whose Length is half the *Latus Rectum*, is equal to the Time of two Oscillations in the Cycloid OST , or to the Time of a Revolution in the Circumference GLD , or in the Circumference HME . *Q. E. D.*

Cor. HENCE if a moving Body is carried in the Circumference of a Circle with the Celerity that is acquired by falling through $\frac{1}{4}$ of the Diameter, the Time of a Revolution will be equal to the Time of two the least Oscillations of a Pendulum, whose Length is the Semi-diameter of the Circle.

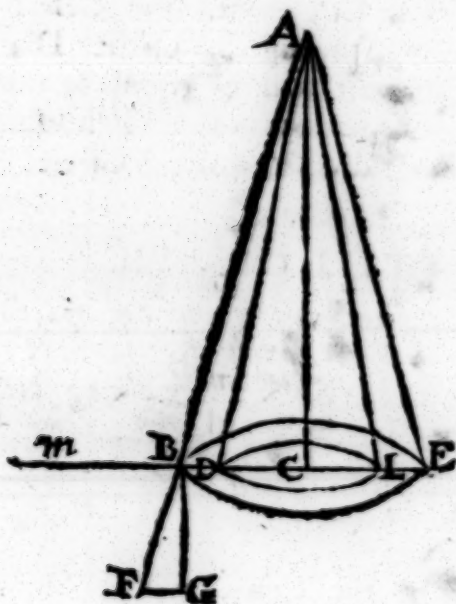
THEOR. VII.

IF two Bodies, suspended by unequal Strings, do so revolve, that they run over Circumferences parallel to the Horizon, the Top of the Strings remaining fixed, but the Heights of the Cones, whose Superficies are described by this Motion, shall be equal; the Times also of the Revolutions will be equal.

LET ABE be the Cone, whose Superficies the String AB describes; also ADL the Cone whose Superficies is described by the String AD : and let C be the Center of the Base of both Cones, and AC their common Height. Now let the moving Body B be consider'd as drawn by three Powers mutually equivalent to each other: whereof one, which is the Force of Gravity, draws the moving Body in the right Line BG perpendicular to the Plane of the Horizon, the other acting according to the Direction Bm , is the Centrifugal Force whereby the moving Body endeavours to recede from the Center

298 *M. Huygens's Theorems, &c.*

Center C of its Orbit; but the third, which is equivalent to and resists the other two, is the contrary Endeavour of the String acting according to the Direction AB; for the Tension of a String is instead of a contrary Power, and in this case performs the same effect. If therefore BF represents the Action of the String, the Centrifugal Force of the moving



Body, and its Force of Gravity, will be expounded by the right Lines FG and BG, [by Theor. 28. Lect. 14.] that is, the Centrifugal Force of the moving Body B, will be to the Force of Gravity as FG to BG, or (by reason the Triangles FBG, ABC are equi-angular) as BC to CA. After the same manner, the Force of Gravity will be to the Centrifugal Force of the moving Body D, as AC to DC: wherefore, by Equality, the Centrifugal Force of the moving Body B, will be to the Centrifugal

fugal Force of the moving Body D, as BC to DC; that is, the Centrifugal Forces are as the Semi-diameters of the Circles, whose Circumferences the moving Bodies describe, and consequently [by Cor. Theor. I.] the Times of the Revolutions are equal. Q. E. D.

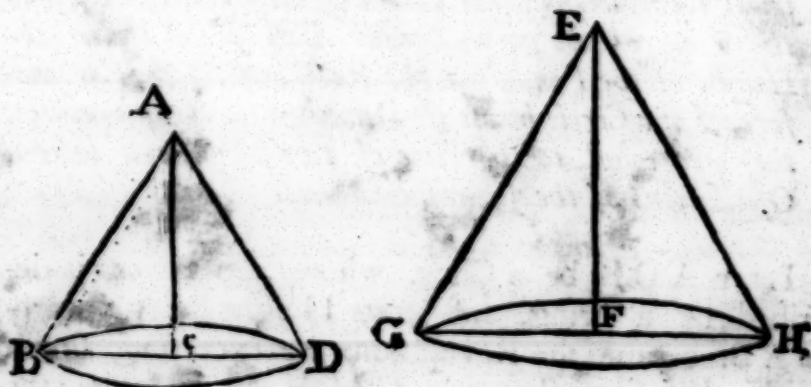
Cor. HENCE the Centrifugal Force is to the Force of Gravity as the Semi-diameter of the Base of the Cone to the Height of the Cone.

Note, BY the Force of Gravity and the Centrifugal Force in this Demonstration, we understand the accelerating Forces of the moving Bodies, unless those Bodies are supposed equal; in which case may be also taken the absolute Forces.

THEOR. VIII.

IF two Bodies, as before, revolve with a Conical Motion, suspended by equal or unequal Strings, and the Heights of the Cones are unequal, the Times of the Revolutions will be in a subduplicate Ratio of their Heights.

LET B and G be the two moving Bodies, and let first of all the Cones ABD, EGH, whose Super-



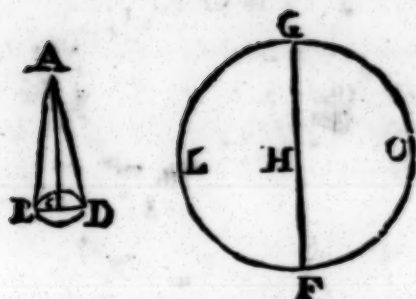
300 *M. Huygens's Theorems, &c.*

ficies are described by the Strings, be similar; [by Corol. Theor. 7.] the Centrifugal Force of the moving Body B will be to the Force of Gravity as BC to AC; and the Centrifugal Force of the moving Body G will be to the same Force of Gravity as GF to FE. But by reason the Triangles ABC, GEF, are equi-angular, BC is to AC as GF to FE: wherefore the Centrifugal Force of the moving Body B will be to the Force of Gravity, as the Centrifugal Force of the moving Body G to the same Force of Gravity, and consequently these Centrifugal Forces will be equal. Therefore, [by Theor. 4.] the Times of the Revolutions of the moving Bodies will be in a subduplicate Ratio of the Semi-diameters; that is, by reason the Triangles ABC, EGF, are equi-angular, in a subduplicate Ratio of the Heights AC and EF. But whatever the Cones are, which the Strings describe, so that their Heights remain unaltered, the Times of the Revolutions will remain unchanged: wherefore in every case the Truth of this Theorem is manifest. Q. E. D.

THEOR. IX.

If a Pendulum carried in a Conical Motion makes very small Revolutions; the Times of each of them have to the Time of the perpendicular Fall from twice the Height of the Pendulum, the Ratio, which the Circumference of a Circle has to its Diameter: and consequently, they are equal to the Time of two very small lateral Oscillations of the same Pendulum.

LET ADB be a Cone, whose Superficies is described by a String, and let its Height be Ac nearly $= AB$, because the Revolutions are exceeding small. With the Semi-diameter $GH = Ac$ describe the Circle



Circle GLFO, and let a Body be supposed to revolve in its Circumference, with the Celerity that is acquired by falling through $\frac{1}{2}$ of its Diameter or $\frac{1}{2}$ D. [By Theor. 5.] its Centrifugal Force will be equal to its Gravity; but the Centrifugal Force of the moving Body B is to the Force of Gravity, and consequently to the Centrifugal Force of the moving Body carried in the Circumference GLF, as Bc to Ac or GH: wherefore the moving Bodies B and G, since their Centrifugal Forces are as the *Radii*, will have equal Times of Revolutions, [by Cor. Theor. 1.] But the Time of Descent through GF or D, is to the Time of Descent through $\frac{1}{2}$ D as D to $\frac{1}{2}$ D, [by Cor. Theor. 12 Left. 11.] and the Time of Descent through $\frac{1}{2}$ D is to the Time of a Revolution in the Circumference GLG, as $\frac{1}{2}$ D to P. Wherefore, by Equality, the Time of Descent through D will be to the Time of a Revolution in the Circumference GLF, or to the Time of a Revolution of the Pendulum ABcD as D to P. The latter part of this Theorem is manifest from the Corollary of Theor. 6.

Cor. HENCE since the Time of the perpendicular Fall is in a subduplicate Ratio of the Space passed over by a falling heavy Body, the Time of Descent from

302 *M. Huygens's Theorems, &c.*

from the Height of a Pendulum will be to the Time of an exceeding small Revolution as $D \times \sqrt{\frac{1}{2}}$ to P.

THEOR. X.

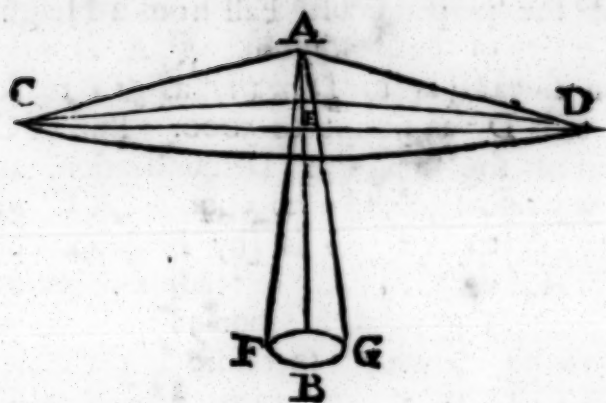
If a moving Body is carried in a Circumference, and performs each Revolution in the Time wherein a Pendulum, having its Length equal to the Semi-diameter of that Circumference, shall finish a very small Revolution in a Conical Motion, or twice an exceeding small lateral Oscillation; that moving Body will have a Centrifugal Force equal to its Gravity.

BECAUSE the moving Bodies B, G, (by Hypothesis) perform their Revolutions in equal Times, [see Fig. of the preceding Theor.] the Centrifugal Force of the moving Body B will be to the Centrifugal Force of the moving Body G, as BC to GH, or BC to AC; but it is as BC to AC, so the Centrifugal Force of the moving Body B to the Force of Gravity, [by Cor. Theor. 7.] wherefore [by 9 El. 5.] the Centrifugal Force of the moving Body G will be equal to the Force of Gravity. Q. E. D.

THEOR. XI.

THE Times of the Revolutions of any Pendulum carried with a Conical Motion, will be equal to the Time of the perpendicular Fall from a Height equal to the String of the Pendulum; when the Angle of Inclination of the String to the Plane of the Horizon will be nearly 2 degr. 54 min. But exactly, if the Sine of that Angle shall be to Radius as a Square inscribed in a Circle to the Square of its Circumference.

LET the Pendulum, whose String describes the Conical Superficies CAD, be such, as the Sine of the



the Angle ACE shall be to Radius (that is, AE to AC) as $\frac{1}{2} D^2$ to P^2 . Let also AFG be the Superficies of a Cone, which the String of the Pendulum describes carried with an exceeding small Motion, whose Height therefore is $AB = AF = AC$. The Time of a Revolution of a moving Body F, will be [by Theor. 8.] to the Time of a Revolution of the moving Body C, in a subduplicate Ratio of AB, or AC to AE: but it is as AC to AE, so (by Hypothesis) P^2 to $\frac{1}{2} D^2$; wherefore the Time of a Revolution of the moving Body F, will be to the Time of a Revolution of the moving Body C, in a subduplicate Ratio of P^2 to $\frac{1}{2} D^2$, that is, in the Ratio of P to $D \times \sqrt{\frac{1}{2}}$. But it is as P to $D \times \sqrt{\frac{1}{2}}$, so [by Cor. Theor. 9.] the Time of the least Revolution, that is, the Time of a Revolution of the Body F, to the perpendicular Fall from the Height of the Pendulum: wherefore the Time of the Revolution of the moving Body F, has the same Proportion to the Time of the Revolution of the moving Body C, as it has to the Time of the perpendicular Fall from a Height equal to the Length of the Pendulum; and con-

304 *M. Huygens's Theorems, &c.*

consequently [by 9 *El.* 4.] the Time of the Revolution of the moving Body C, will be equal to the Time of the perpendicular Fall from a Height equal to the Length of the Pendulum. *Q. E. D.*

BUT since P is to D nearly as 314 to 100, P^2 will be to $\frac{1}{4} D^2$ as 98596 to 5000. But AC is to AE, from the foregoing Demonstration, as P^2 to $\frac{1}{4} D^2$; wherefore 98596 is to 5000 as AC to AE; and as AC to AE, so is [by Trigonometry] the Sine of the Angle AEC, or Radius 100000 to the Sine of the Angle ACE: but 98596 is to 5000 as 100000 to 5070, which therefore is the Sine of the Angle ACE, to which nearly answer 2 Degrees 54 Minutes.

THEOR. XII.

IF two Pendulums equal in Weight, but having their Strings unequal, do revolve in a Conical Motion, and the Heights of the Cones are equal, the Forces wherewith they stretch their Strings, will be in the same Ratio, as is that of the Length of the Strings.

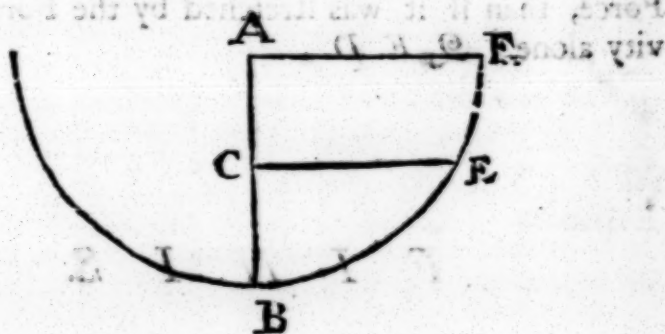
THIS is manifest from Theor. 7. For the Force of Gravity in both the Cones, is to the Tension of the String, as the Height of the Cone to the Length of the String; and since the Height of the Cones is the same, it is evident that the Tensions of the Strings are proportionable to their Lengths. *Q. E. D.*

THEOR. XIII.

IF a simple Pendulum is moved with the greatest lateral Oscillation, that is, if it descends through the whole Quadrant of a Circle, when it shall arrive at the lowest Point of the Circumference, it will draw its
String

String with triple a greater Force, than if it was only simply suspended by it.

LET AB be a Pendulum moved through the Quadrant FB , bisect AB in C , through which draw CE perpendicular to AB , and meeting the Circumference in E . If the Pendulum descended



only through the Arch EB , it would acquire in the Point B the same Velocity, as if it had descended through $CB = \frac{1}{2}$ of the Diameter [by Corol. 1. Prop. 38. of the 15th Lect.] so that [by Theor. 5.] it will have in the Point B a Centrifugal Force equal to its Gravity: and consequently the Gravity and the Centrifugal Force joined together, will draw the String with twice a greater Force than it would be drawn by the Gravity alone. But if the Pendulum is raised to F , after its Descent to B , it will acquire the same Velocity, as if it had fallen through AB . But AB is to BC in a duplicate Ratio of the Velocity acquired in the Descent through AB , to the Velocity acquired in the Descent through BC ; wherefore also AB will be to BC [by Theor. 3.] as the Centrifugal Force of the moving Body in the Point B after the Descent through FB , to the Centrifugal Force in the Point B , after the Descent only through EB . So that

306 *M. Huygens's Theorems, &c.*

the Centrifugal Force of the moving Body after the Descent through F B, will be double the Centrifugal Force after the Descent through E B; that is, the Centrifugal Force in the Point B, after the Fall through F B, will be double the Force of Gravity. Wherefore the String, by the Centrifugal Force and the Force of Gravity acting together, and according to the same Direction, is drawn by triple a greater Force, than if it was stretched by the Force of Gravity alone. Q. E. D.

F I N I S



BOOKS

**BOOKS printed for M. SENEX, W. INNIS, T.
LONGMAN and T. SHEWELL.**

1. **A** New Mathematical Dictionary: Wherein is contain'd, not only the Explanation of the bare Terms, but likewise an History of the Rise, Progress, State, Properties, &c. of things, both in pure Mathematicks, and Natural Philosophy, so far as it comes under a Mathematical Consideration. By E. Stone, F. R. S. Octavo. Second Edition.
2. Mathematical Elements of Natural Philosophy confirm'd by Experiments; or, An Introduction to Sir Isaac Newton's Philosophy, Written in Latin by William James s'Gravesande, Doctor of Laws and Philosophy, Professor of Mathematicks and Astronomy at Leyden, and F. R. S. at London. Translated into English by J. T. Desaguliers, LL. D. F. R. S. and Chaplain to his Grace the Duke of Chandos. in two vol. Octavo.
3. An Essay on Perspective. By W. J. s'Gravesande, &c. Octavo, 1724.
4. The Religious Philosopher; or the right Use of contemplating the Works of the Creator: 1st, In the wonderful Structure of animal Bodies, and in particular Man. 2^{dly}, In the no less wonderful and wise Formation of the Elements, their various Effects upon animal and vegetable Bodies. And, 3^{dly}, In the most amazing Structure of the Heavens, with all its Furniture, design'd for the Conviction of Atheists and Infidels, by that learned Mathematician, Dr. Nieuventys. To which is prefix'd a Letter to the Translator, by the Reverend J. T. Desaguliers, LL. D. F. R. S. The Fourth Edition, adorned with Cuts. Quarto, 1745.
5. An Analytick Treatise of Conic Sections, and their Use, for resolving of Equations in determinate and indeterminate Problems, being the posthumous Work of the Marquis de l'Hospital. Quarto, 1723.
6. Universal Arithmetick; or a Treatise of Arithmetical Composition and Resolution: To which is added, Dr. Halley's Method of finding the Roots of Equations arithmetically. Translated from the Latin, by the late Mr. Raphson, and revised and corrected by Mr. Cunn, 8vo.
7. The Astronomical Principles of Religion, natural and reveal'd, containing a particular Account of the System of the Universe, and the Truth of that System demonstrated from the established Laws of Matter and Motion; together with probable Conjectures of the Nature and Uses of the several celestial Bodies, contain'd in the same System, &c. By Will. Whiston, sometime Professor of the Mathematicks in the University of Cambridge.

8. The Philosophical Works of the Honourable Robert Boyle, Esq; abridg'd, methodiz'd, and dispos'd under the several Heads of Physics, Statics, Pneumatics, Natural History, Chymistry and Medicine. The whole illustrated with Notes, containing the Improvements made in the several Parts of natural and experimental Knowledge, since his Time, in three vol. with Cuts. By Peter Shaw. M. D. Quarto. Second Edition.

9. Philosophiæ Naturalis Principia Mathematica. Auctore *Isaaco Newtono* Equite Aurato. Editio tertia, aucta & emendata. Quarto, Lond. 1716.

10. Epistola ad Amicum de *Cotesij* Inventis Curvarum ratione, quæ cum Circulo & Hyperbola comparisonem admittunt; cui additur Appendix, Auctore *Hen. Pemberton*, M. D. R. S. S. Quarto, 1721.

11. Elementa Arithmeticæ Numerosæ & Speciosæ in usum Juventutis Academicæ, Auctore *Edw. Wells*, S. T. P. Editio altera Auctior, 8vo, 1726.

12. Opticks: or a Treatise of the Reflexions, Refractions, Inflexions, and Colours of Light. The third Edition corrected. By *Sir Isaac Newton*, Kt. 8vo.

13. Optice, sive de Reflexionibus, Refractionibus, Inflexionibus & Coloribus Lucis, Libri tres. Authore *Isaaco Newton*, Equite Aurato. Latine reddidit *Samuel Clarke*, S. T. P. Editio secunda auctior, 8vo, 1719.

14. Introductio ad Veram Physicam. Authore *Joanne Keill*, M. D. Astr. Prof. Sav. R. S. S. Editio quarta, 8vo, 1719.

15. Geometria organica, sive Descriptio Linearum curvarum universalis. Auctore *Colino Mac-Laurin* Matheseos in Collegio novo Abredonensi Professore, & Reg. Soc. Socio. Quarto.

16. Philosophical Transactions; giving an Account of the present Undertakings, Studies and Labours of the Ingenious in many considerable Parts of the World, from 1665 to 1726. In 33 volumes. Continued and published by *Dr. James Jurin*, Secretary to the Royal Society. N. B. There are 392 Numbers, most of which may be had separate.

There is now made by *Mary Senex*, a large Pair of Globes of 28 Inches Diameter, from the latest Observations and Discoveries, fit to adorn the Librarys of the Curious. Also Globes of 17, 12, and 9 Inches Diameter; *Dr. Halley's* Zodiac, *Mr. Whiston's* solar System, and a Pair of large Hemispheres, containing all the Stars in *Mr. Flamsteed's* Catalogue. These, with the best Maps of the World, and Quarters, and all the general Divisions of *Europe*, are sold by her, at the Globe, over-against *St. Dunstan's* Church, in *Fleet-street*.

